Correlation of Quadratic Boolean Functions: Cryptanalysis of All Versions of Full MORUS

Siwei Sun

Joint work with:
Danping Shi Yu Sasaki Chaoyun Li Lei Hu

Chinese Academy of Sciences, China

NTT Secure Platform Laboratories, Japan

imec-COSIC, Dept. Electrical Engineering (ESAT), KU Leuven, Belgium

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Outlines

1. Correlation and Linear Cryptanalysis
2. Correlation of Quadratic Boolean Functions
3. Cryptanalysis of MORUS
4. Conclusion and Discussion

Outline

Correlation

Let $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$ be a Boolean function with ANF

$$f(\boldsymbol{x}) = \sum_{\boldsymbol{u} \in \mathbb{F}_2^n} a_{\boldsymbol{u}} \boldsymbol{x}^{\boldsymbol{u}},$$

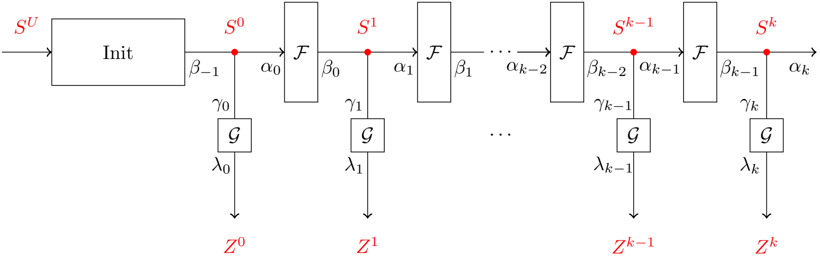
where $\boldsymbol{x} = (x_1, \cdots, x_n)$, $\boldsymbol{u} = (u_1, \cdots, u_n)$, $a_{\boldsymbol{u}} \in \mathbb{F}_2$, and $\boldsymbol{x}^{\boldsymbol{u}} = \prod_{i=1}^{n} x_i^{u_i}$.

Definition (Correlation)

The correlation of an n-variable Boolean function f is $\mathrm{cor}(f) = \frac{1}{2^n} \sum_{\boldsymbol{x} \in \mathbb{F}_2^n} (-1)^{f(\boldsymbol{x})}$, and the weight of the correlation is defined as $-\log_2 |\mathrm{cor}(f)|$.

- $Pr(f = 0) = \frac{1}{2} + \frac{1}{2}\mathrm{cor}(f)$

Linear Cryptanalysis



Object: $\max \left|\text{cor}\left(\sum_{i=0}^{k} \lambda_i Z^i\right)\right|$

Note that $\sum_{i=0}^{k} \lambda_i Z^i$ is a Boolean function whose variables are bits of S^0.

Definition (Correlation)

The correlation of an n-variable Boolean function f is $\mathrm{cor}(f) = \frac{1}{2^n} \sum_{\boldsymbol{x} \in \mathbb{F}_2^n} (-1)^{f(\boldsymbol{x})}$, and the weight of the correlation is defined as $-\log_2 |\mathrm{cor}(f)|$.

- Brute force the input
- Graph-based method [TIM$^+$18]
- ... ...

Outline

Definition (Disjoint Quadratic Boolean Function)

A quadratic Boolean function $f(x_1, \cdots, x_n)$ is disjoint if no variable x_i appears in more than one quadratic term.

Example

$$x_1x_2 + x_3x_4$$

$$x_1x_3 + x_2x_4 + x_2 + x_5$$

Counter-Example

$$x_1x_2 + x_2x_3$$

lemma

Let $f = x_{i_1}x_{i_2} + \cdots + x_{i_{2k-1}}x_{i_{2k}} + x_{j_1} + \cdots + x_{j_s}$ be a disjoint quadratic Boolean function. Then the correlation of f is

$$\begin{cases} (-1)^{\sum_{t=1}^{k} \mathrm{Coe}_f(x_{i_{2t-1}})\mathrm{Coe}_f(x_{i_{2t}})} \cdot 2^{-k} & \{j_1, \cdots, j_s\} \subseteq \{i_1, \cdots, i_{2k}\} \\ 0 & \{j_1, \cdots, j_s\} \subsetneq \{i_1, \cdots, i_{2k}\} \end{cases}$$

where $\mathrm{Coe}_f(\boldsymbol{x}^{\boldsymbol{u}})$ denotes the coefficient of the monomial $\boldsymbol{x}^{\boldsymbol{u}}$ in the ANF of f.

Examples

$$|\mathrm{cor}(x_1x_2 + x_3x_4)| = 2^{-2}$$

$$|\mathrm{cor}(x_1x_3 + x_2x_4 + x_2 + x_5)| = 0$$

$$|\mathrm{cor}(x_1x_3 + x_2x_4 + x_2 + x_3)| = 2^{-2}$$

Idea

Given a quadratic Boolean function, transform it into a disjoint quadratic Boolean function such that the transformation is correlation invariant (up to a minus sign).

Example

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

Example

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

$f = x_1{\color{red}x_2} + x_1x_5 + {\color{red}x_2}x_3 + {\color{red}x_2}x_4 + x_1 + x_2$

Example

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

$f = x_1{\color{red}x_2} + x_1x_5 + {\color{red}x_2}x_3 + {\color{red}x_2}x_4 + x_1 + x_2$

$f = {\color{red}x_2}(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$

Example

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

$f = x_2(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$

$x_1 \leftarrow x_1 + x_3 + x_4$

$x_j \leftarrow x_j, \quad j \neq 1$

Example

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

$f = x_2(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$

$x_1 \leftarrow x_1 + x_3 + x_4$

$x_j \leftarrow x_j, \quad j \neq 1$

$f = x_1x_2 + x_1x_5 + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$

Example

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$

$f = x_2(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$

$x_1 \leftarrow x_1 + x_3 + x_4$

$x_j \leftarrow x_j, \quad j \neq 1$

$f = x_1x_2 + x_1x_5 + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$

Example

$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$

$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$

$$f = x_2(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$$

$$x_1 \leftarrow x_1 + x_3 + x_4$$

$$x_j \leftarrow x_j, \quad j \neq 1$$

$$f = x_1x_2 + x_1x_5 + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$

$$f = x_1(x_2 + x_5) + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$

Example

$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$
$$f = x_1\textcolor{red}{x_2} + x_1x_5 + \textcolor{red}{x_2}x_3 + \textcolor{red}{x_2}x_4 + x_1 + x_2$$
$$f = \textcolor{red}{x_2}(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$$
$$x_1 \leftarrow x_1 + x_3 + x_4$$
$$x_j \leftarrow x_j, \quad j \neq 1$$
$$f = \textcolor{red}{x_1}x_2 + \textcolor{red}{x_1}x_5 + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$f = \textcolor{red}{x_1}(x_2 + x_5) + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$x_2 \leftarrow x_2 + x_5$$
$$x_j \leftarrow x_j, \quad j \neq 2$$

Example

$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$
$$f = x_1{\color{red}x_2} + x_1x_5 + {\color{red}x_2}x_3 + {\color{red}x_2}x_4 + x_1 + x_2$$
$$f = {\color{red}x_2}(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$$
$$x_1 \leftarrow x_1 + x_3 + x_4$$
$$x_j \leftarrow x_j, \quad j \neq 1$$
$$f = {\color{red}x_1}x_2 + {\color{red}x_1}x_5 + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$f = {\color{red}x_1}(x_2 + x_5) + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$x_2 \leftarrow x_2 + x_5$$
$$x_j \leftarrow x_j, \quad j \neq 2$$
$$f = x_1x_2 + x_3x_5 + x_4x_5 + x_1 + x_2 + x_3 + x_4 + x_5$$

Example

$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$
$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$
$$f = x_2(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$$
$$x_1 \leftarrow x_1 + x_3 + x_4$$
$$x_j \leftarrow x_j, \quad j \neq 1$$
$$f = x_1x_2 + x_1x_5 + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$f = x_1(x_2 + x_5) + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$x_2 \leftarrow x_2 + x_5$$
$$x_j \leftarrow x_j, \quad j \neq 2$$
$$f = x_1x_2 + x_3x_5 + x_4x_5 + x_1 + x_2 + x_3 + x_4 + x_5$$

Example

$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$
$$f = x_1\color{red}{x_2} + x_1x_5 + \color{red}{x_2}x_3 + \color{red}{x_2}x_4 + x_1 + x_2$$
$$f = \color{red}{x_2}(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$$
$$x_1 \leftarrow x_1 + x_3 + x_4$$
$$x_j \leftarrow x_j, \quad j \neq 1$$
$$f = \color{red}{x_1}x_2 + \color{red}{x_1}x_5 + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$f = \color{red}{x_1}(x_2 + x_5) + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$x_2 \leftarrow x_2 + x_5$$
$$x_j \leftarrow x_j, \quad j \neq 2$$
$$f = \color{blue}{x_1x_2} + x_3\color{red}{x_5} + x_4\color{red}{x_5} + x_1 + x_2 + x_3 + x_4 + x_5$$
$$f = \color{blue}{x_1x_2} + \color{red}{x_5}(x_3 + x_4) + x_1 + x_2 + x_3 + x_4 + x_5$$

Example

$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$
$$f = x_1x_2 + x_1x_5 + x_2x_3 + x_2x_4 + x_1 + x_2$$
$$f = x_2(x_1 + x_3 + x_4) + x_1x_5 + x_1 + x_2$$
$$x_1 \leftarrow x_1 + x_3 + x_4$$
$$x_j \leftarrow x_j, \quad j \neq 1$$
$$f = x_1x_2 + x_1x_5 + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$f = x_1(x_2 + x_5) + x_3x_5 + x_4x_5 + x_1 + x_3 + x_4 + x_2$$
$$x_2 \leftarrow x_2 + x_5$$
$$x_j \leftarrow x_j, \quad j \neq 2$$
$$f = x_1x_2 + x_3x_5 + x_4x_5 + x_1 + x_2 + x_3 + x_4 + x_5$$
$$f = x_1x_2 + x_5(x_3 + x_4) + x_1 + x_2 + x_3 + x_4 + x_5$$
$$x_3 \leftarrow x_3 + x_4$$
$$x_j \leftarrow x_j, \quad j \neq 3$$
$$f \leftarrow x_1x_2 + x_3x_5 + x_1 + x_2 + x_3 + x_5$$

Theorem

Given a quadratic boolean function $f(\boldsymbol{x}) = f(x_1, \cdots, x_n)$, the algorithm outputs a disjoint quadratic Boolean function $\hat{f}(\boldsymbol{x})$ and an invertible $n \times n$ matrix M, such that $\hat{f}(\boldsymbol{x}) = f(\boldsymbol{x}M)$. Moreover, The algorithm has time complexity $\mathcal{O}(n^{3.8})$ and memory complexity $\Omega(n^2)$.

Remark

On 22-06-2019, we received an E-mail from Ryan Williams (MIT), which indicated that essentially the same theory concerning quadratic forms had been developed much earlier (despite some superficial differences in the appearance).

- Leonard Carlitz: *Gauss sums over finite fields of order* 2^n. Acta Arithmetica. 1969.
- Andrzej Ehrenfeucht and Marek Karpinski: *The computational complexity of (xor, and)-counting problems*. International Computer Science Inst. 1990
- Roland Mirwald and Claus-Peter Schnorr: *The Multiplicative Complexity of Quadratic Boolean Forms*. Theor. Comput. Sci. 1992.

Outline

The CAESAR Competition

- R1: 58 candidates, 2014.3-2015.7
- R2: 29 candidates, 2015.7-2016.8
- R3: 15 candidates, 2016.8-2018.3
- RF: 7 candidates, 2018.3-2019.3

Finalists of CAESAR

Lightweight applications

- ACORN
- ASCON

High-performance applications

- AEGIS
- OCB
- MORUS

Defense in depth

- COLM
- Deoxys-II

6 winners were announce on March 20, 2019.

MORUS

- Designers: Hongjun Wu and Tao Huang
- Stream-cipher like design
- MORUS-640, 128-bit key
- MORUS-1280, 128-bit or 256-bit key
- MORUS-1280-256 was broken in ASIACRYPT 2018 [AEL+18]

Name	State size ($5q$)	Register size (q)	Word size ($q/4$)	Key size
MORUS-640-128	640	128	32	128
MORUS-1280-128	1280	256	64	128
MORUS-1280-256	1280	256	64	256

Encryption Algorithm

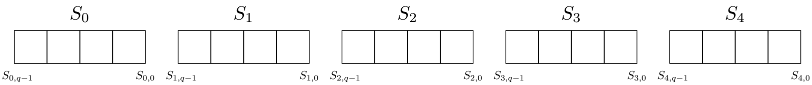


Figure: Internal State

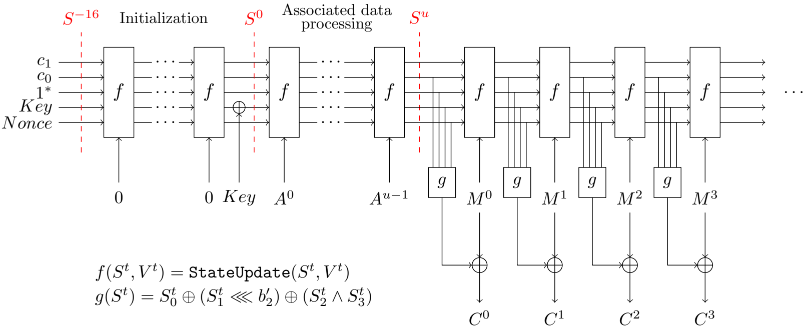
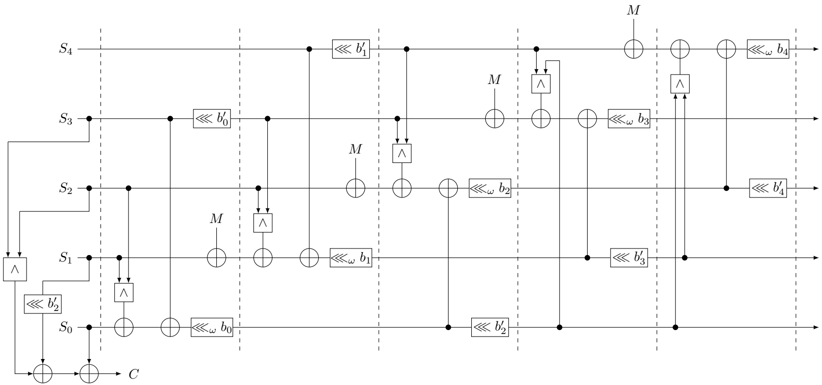
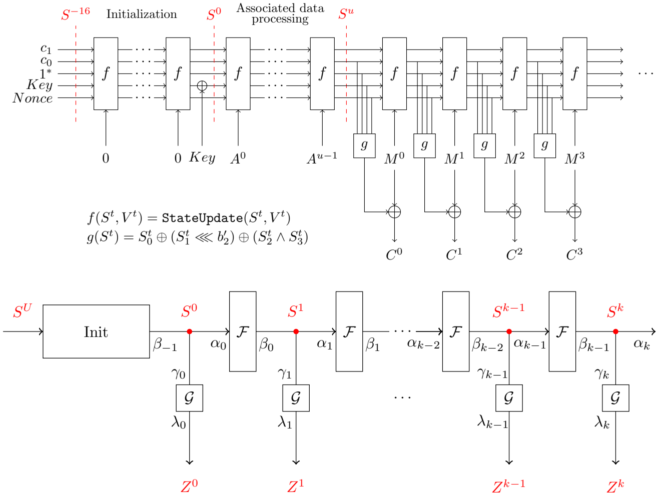


Figure: The encryption algorithm of MORUS

State Update Function

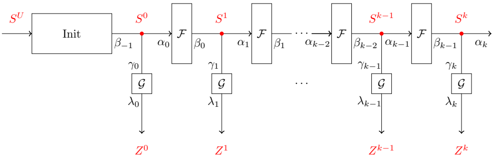


b' is multiple of word size



S^{-16}
Initialization
S^0
Associated data processing
S^u
c_1
c_0
1^*
Key
$Nonce$
f
g
0
0
Key
A^0
A^{u-1}
M^0
M^1
M^2
M^3
C^0
C^1
C^2
C^3
$f(S^t, V^t) = \texttt{StateUpdate}(S^t, V^t)$
$g(S^t) = S_0^t \oplus (S_1^t \lll b_2') \oplus (S_2^t \wedge S_3^t)$
S^U
Init
S^0
S^1
S^{k-1}
S^k
$\mathcal{F}$
$\mathcal{G}$
β_{-1}
α_0
β_0
α_1
β_1
α_{k-2}
β_{k-2}
α_{k-1}
β_{k-1}
α_k
γ_0
γ_1
γ_{k-1}
γ_k
λ_0
λ_1
λ_{k-1}
λ_k
Z^0
Z^1
Z^{k-1}
Z^k

- For a block cipher, we have many tools (Matsui's branch and bound, MILP, SAT, SMT, CP etc.) to search for its linear trails.
- For the key stream generator?



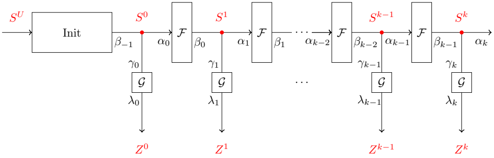
Definition

linear trail A linear trail of the key stream generator shown in Fig:

$$(\beta_{-1}, \gamma_0, \lambda_0, \alpha_0, \beta_0, \cdots, \alpha_{k-1}, \beta_{k-1}, \gamma_k, \lambda_k, \alpha_k)$$

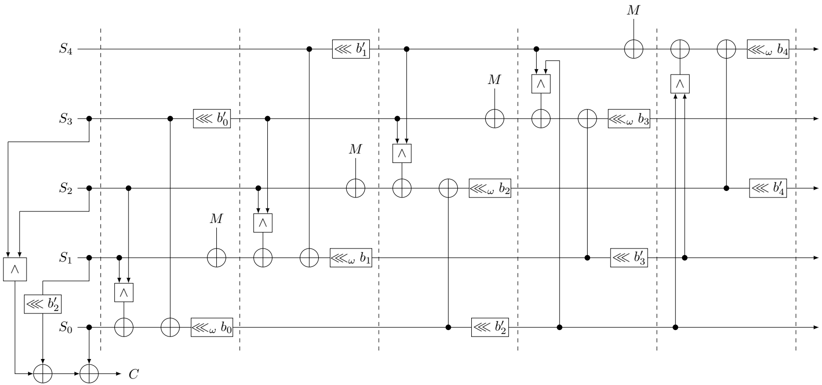
is said to be exploitable if and only if $\beta_{-1} = 0$, $\alpha_k = 0$, and $\alpha_i \oplus \gamma_i \oplus \beta_{i-1} = 0$ for $0 \le i \le k$.

Linear characteristic



$$\begin{cases} \beta_{-1} = 0 \\ \alpha_k = 0 \\ \alpha_i + \gamma_i + \beta_{i-1} = 0, & 0 \leq i \leq k \\ \gamma_i S^i + \lambda_i Z^i = 0, & 0 \leq i \leq k \\ \alpha_i S^i + \beta_i S^{i+1} = 0, & 0 \leq i \leq k-1 \end{cases} \tag{1}$$

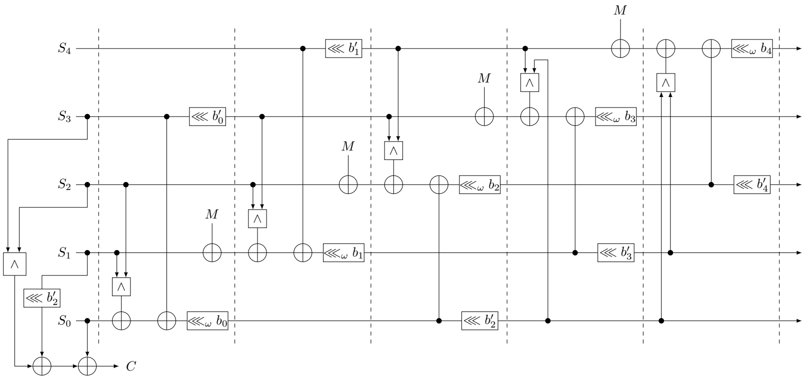
Rotationally Invariant Masks [AEL+18]



b' is multiple of word size

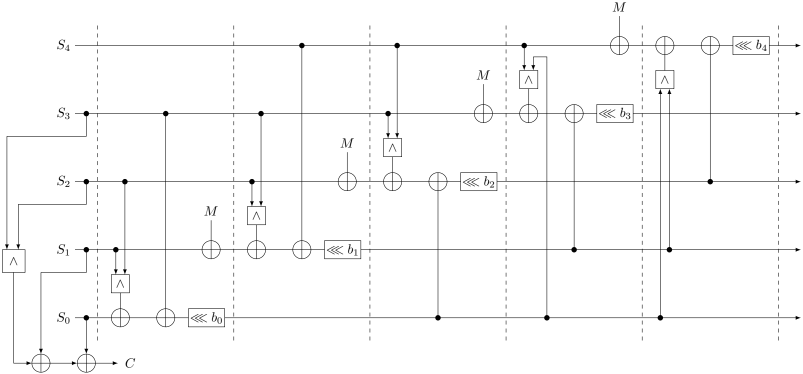
- MiniMORUS: each register contains a single word

MiniMORUS



S_4
S_3
S_2
S_1
S_0
M
M
M
M
$\lll b_0'$
$\lll b_1'$
$\lll b_2'$
$\lll b_3'$
$\lll b_4'$
$\lll_\omega b_0$
$\lll_\omega b_1$
$\lll_\omega b_2$
$\lll_\omega b_3$
$\lll_\omega b_4$
C

MiniMORUS



- Any linear characteristic search tool (Matsui, MILP, SAT/SMT, CP, etc.) can be applied.
- The resulting characteristics are only locally sound!
- Any characteristic can be converted to a quadratic boolean function in variables $S_{i,j}^t$, from which the correlation should be recalculated!

$$\begin{cases} f_1(x_1, x_2, x_3) = x_1x_2 + x_2, \mathrm{cor}(f_1) = 2^{-1} \\ f_2(x_1, x_2, x_3) = x_1x_3, \mathrm{cor}(f_2) = 2^{-1} \end{cases}.$$

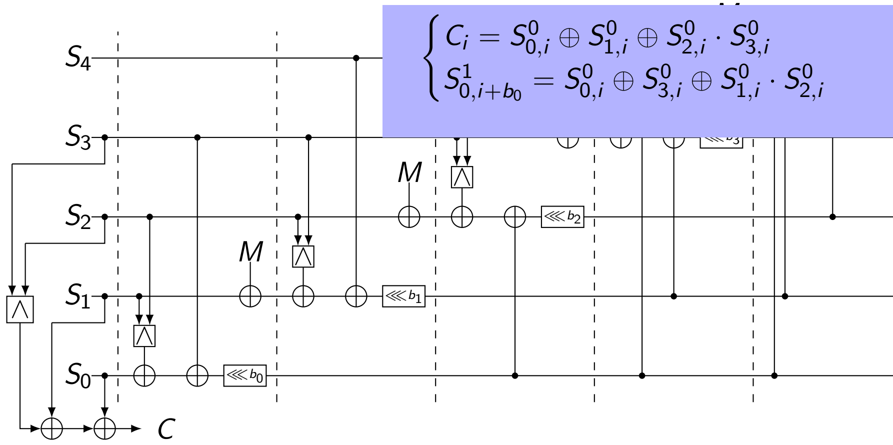
$$f = f_1 + f_2 = x_1x_2 + x_1x_3 + x_2$$

$$\mathrm{cor}(f) = 0 \neq 2^{-2}$$

Table: An invalid trail of MiniMORUS-640 with span 3

Round		Linear masks				
0	α_0	40400000	40400000	00000000	40400000	00000000
		08000008	00400000	00000000	00000000	00000000
		08000008	00200000	00000000	00000000	00400000
		08000008	00200000	00000000	00000000	00400000
		08000008	00200000	00000000	00000000	00400000
	β_0	08000008	00200000	00400000	00000000	00000008
	γ_0	40400000	40400000	00000000	40400000	00000000
	λ_0	40400000				
1	α_1	20600000	28400008	00400000	20600000	00000008
		0c000004	08000008	00000000	00000000	00000008
		0c000004	04000004	08000000	00000000	08000000
		04000004	04000004	00000004	00000000	00000000
		04000004	04000004	00000004	00000000	00000000
	β_1	04000004	04000004	00000004	00000000	00000000
	γ_1	28600008	28600008	00000000	20600000	00000000
	λ_1	28600008				
2	γ_2	04000004	04000004	00000004	00000000	00000000
	λ_2	04000004				

Dependent AND Gates



$$\begin{cases} C_i = S_{0,i}^0 \oplus S_{1,i}^0 \oplus S_{2,i}^0 \cdot S_{3,i}^0 \\ S_{0,i+b_0}^1 = S_{0,i}^0 \oplus S_{3,i}^0 \oplus S_{1,i}^0 \cdot S_{2,i}^0 \end{cases}$$

Table: A linear trail of MiniMORUS-640 with correlation -2^{-8}

Round		Linear masks				
0	α_0	10000000	10000000	00000000	10000000	00000000
		00000002	00000000	00000000	00000000	00000000
		00000002	00000000	00000000	00000000	00000000
		00000002	00000000	00000000	00000000	00000000
		00000002	00000000	00000000	00000000	00000000
	β_0	00000002	00000000	00000000	00000000	00000000
	γ_0	10000000	10000000	00000000	10000000	00000000
	λ_0	10000000				
1	α_1	08000200	08000202	00000002	08000200	00000000
		00004001	00000002	00000002	00000000	00000000
		00004001	00000001	00000000	00000000	00000002
		00004001	00000001	00000000	00000000	00000002
		00004001	00000001	00000000	00000000	00000002
	β_1	00004003	00000003	00000002	00000000	00004000
	γ_1	08000202	08000202	00000002	08000200	00000000
	λ_1	08000202				
2	α_2	00000100	00004100	00000000	00000100	00004000
		00002000	00004000	00000000	00000000	00004000
		00002000	00002000	00000000	00000000	00000000
		00002000	00002000	00000000	00000000	00000000
		00002000	00002000	00000000	00000000	00000000
	β_2	00002000	00002000	00000000	00000000	00000000
	γ_2	00004103	00004103	00000002	00000100	00000000
	λ_2	00004103				
3	γ_3	00002000	00002000	00000000	00000000	00000000
	λ_3	00002000				

- We only list the values for α_i, β_i, γ_i, and λ_i. Actually, for every input and output bits of all the AND gates involved, the solution specifies their masks.
- For every AND gate whose output mask is 1 (active AND gates), we can write down a equation in $S_{i,j}^t$.
- Summing up this equations gives $\sum \lambda_i Z_i$ expressed in a quadratic Boolean function in $S_{i,j}^t$.
- Trails for MiniMORUS can be extended to full MORUS.

Table: A summary of the results

Target	Span	$\lvert\mathrm{cor}\rvert$	Data	Time	Source
MiniMORUS-640	5	2^{-16}	2^{32}	2^{32}	[AEL$^+$18]
	4	2^{-8}	2^{16}	2^{16}	Ours
MiniMORUS-1280	5	2^{-16}	2^{32}	2^{32}	[AEL$^+$18]
	4	2^{-8}	2^{16}	2^{16}	Ours
MORUS-640-128	4	2^{-38}	2^{76}	2^{76}	Ours
MORUS-1280-128	4	2^{-38}	2^{76}	2^{76}	Ours
MORUS-1280-256	5	2^{-76}	2^{152}	2^{152}	[AEL$^+$18]
	4	2^{-38}	2^{76}	2^{76}	Ours

- Distinguishing attack
- Message recovery attack

Assumptions

- S^0 is random (quite reasonable!).
- S^is are independent for different i. (??)

Table: Verification for MiniMORUS

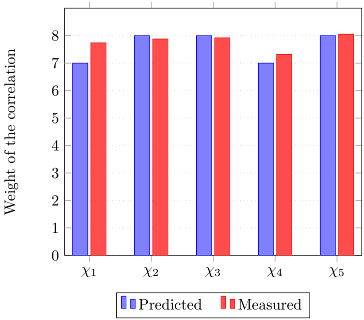
Version	Experiments	Theoretically
MiniMORUS-640	$2^{-7.7919}$	2^{-8}
MiniMORUS-1280	$2^{-8.1528}$	2^{-8}

Table: The five trail fragments of MORUS-640

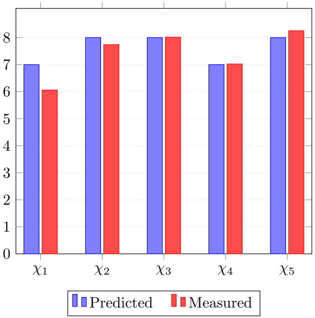
	Trail fragment	Weight
χ_1	$C^0_{\{124,92,60,28\}} \oplus C^1_{\{97,65,33,1\}} = S^1_{4,\{97,65,33,1\}} \oplus S^2_{1,\{96,64,32,0\}}$	7
χ_2	$C^1_{\{123,91,59,27\}} \oplus C^2_{\{96,64,32,0\}} = S^2_{1,\{96,64,32,0\}}$	8
χ_3	$C^2_{\{104,72,40,8\}} \oplus C^3_{\{109,77,45,13\}} = S^3_{1,\{109,77,45,13\}}$	8
χ_4	$C^1_{\{105,73,41,9\}} \oplus C^2_{\{110,78,46,14\}} = S^3_{1,\{109,77,45,13\}} \oplus S^2_{4,\{110,78,46,14\}}$	7
χ_5	$C^2_{\{97,65,33,1\}} = S^1_{4,\{97,65,33,1\}} \oplus S^2_{4,\{110,78,46,14\}}$	8

Table: The five trail fragments of MORUS-1280

	Trail fragment	Weight
χ_1	$C^0_{\{208,144,80,16\}} \oplus C^1_{\{221,157,93,29\}} = S^1_{4,\{221,157,93,29\}} \oplus S^2_{1,\{203,139,75,11\}}$	7
χ_2	$C^1_{\{254,190,126,62\}} \oplus C^2_{\{203,139,75,11\}} = S^2_{1,\{203,139,75,11\}}$	8
χ_3	$C^2_{\{194,130,66,2\}} \oplus C^3_{\{207,143,79,15\}} = S^3_{1,\{207,143,79,15\}}$	8
χ_4	$C^1_{\{212,148,84,20\}} \oplus C^2_{\{225,161,97,33\}} = S^3_{1,\{207,143,79,15\}} \oplus S^2_{4,\{225,161,97,33\}}$	7
χ_5	$C^2_{\{221,157,93,29\}} = S^1_{4,\{221,157,93,29\}} \oplus S^2_{4,\{225,161,97,33\}}$	8



(a) MORUS-640



(b) MORUS-1280

Figure: Experimental verification of the trail fragments of MORUS-640 and MORUS-1280

Outline

- Correlation of quadratic Boolean function can be computed efficiently.
- How about Boolean functions with higher degrees?
- How can we search for trails which are not rotationally invariant?
- MILP based search can only deal with small spans.
- Some manual analysis targeting Trivium, SNOW, and ZUC using very large spans!

Thanks! Any questions?

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