Towards a better understanding of (post-)quantum security of symmetric key schemes

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@ASK 2019 (2019.12.13)

Introduction

Quantum Attacks against Symmetric Cryptosystems?

It has been said that symmetric key schemes would not to be much affected by quantum computers

Known Quantum Attacks：～２０１０

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	Classical	Quantum
Exhaustive Key Search	$O(2^n)$	$O(2^{n/2})$
Collision Finding	$O(2^{n/2})$	$O(2^{n/3})$

"2n-bit key suffices"

Known Quantum Attacks : Today

NTT

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Forgery against CBC-MAC	$O(2^{n/2})$	Polynomial time

Remark：The last two attacks assumes that quantum keyed oracles are available

Quantum Attacks against Symmetric Cryptosystems?

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would not to be much [illegible] by quantum computers

Symmetric key schemes may be significantly affected !!

- Attacks by Kuwakado and Morii at ISIT2010, ISITA2012
- Attacks by Kaplan et al. at CRYPTO2016

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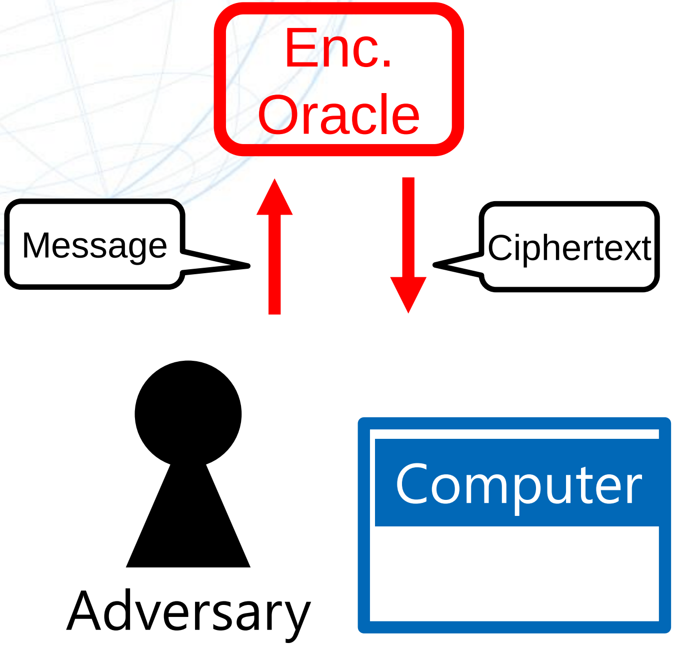
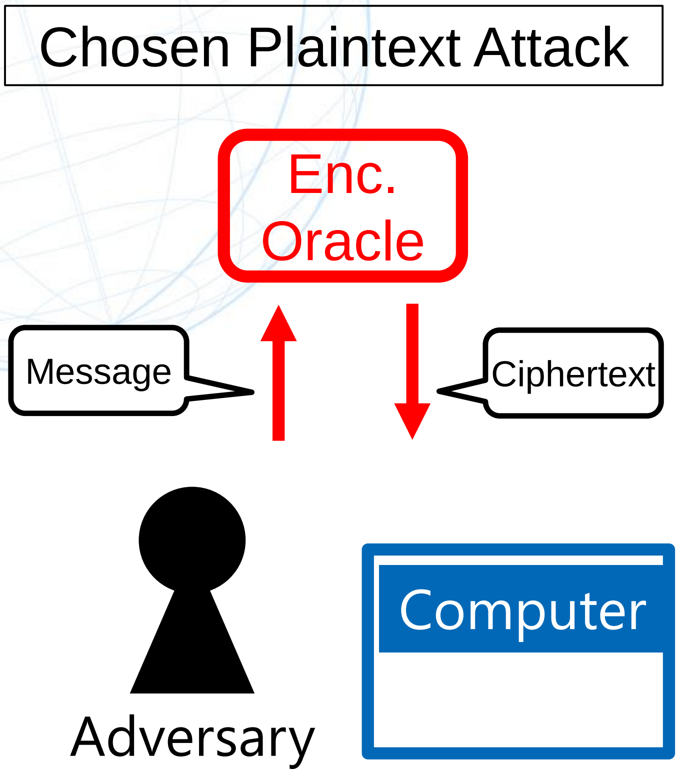
Post-quantum security of symmetric schemes should be analyzed more carefully

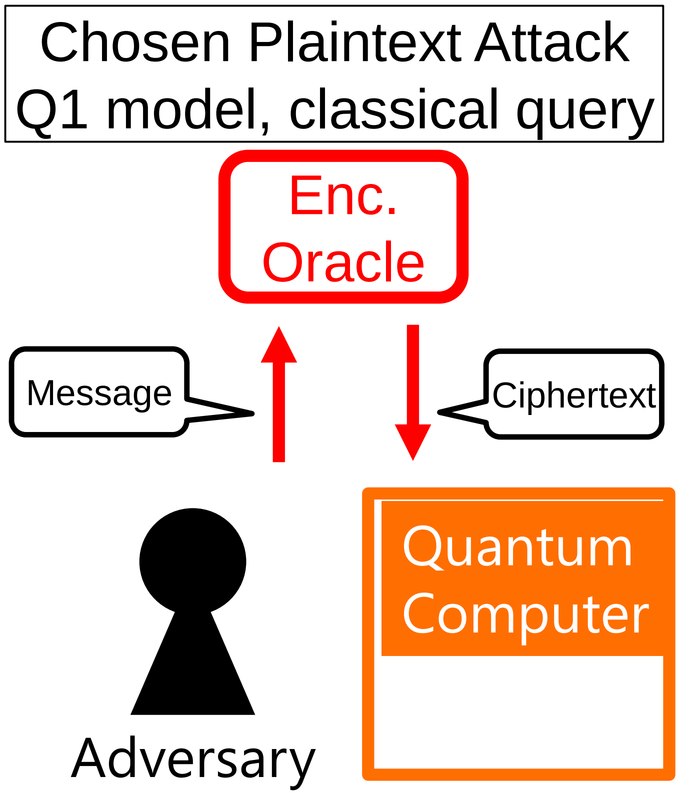
Attack Models

Chosen Plaintext Attack

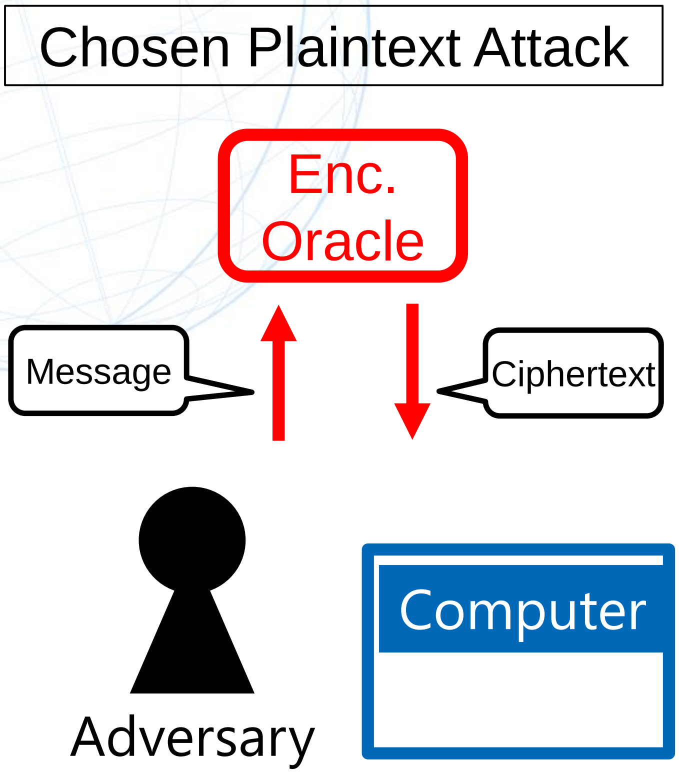


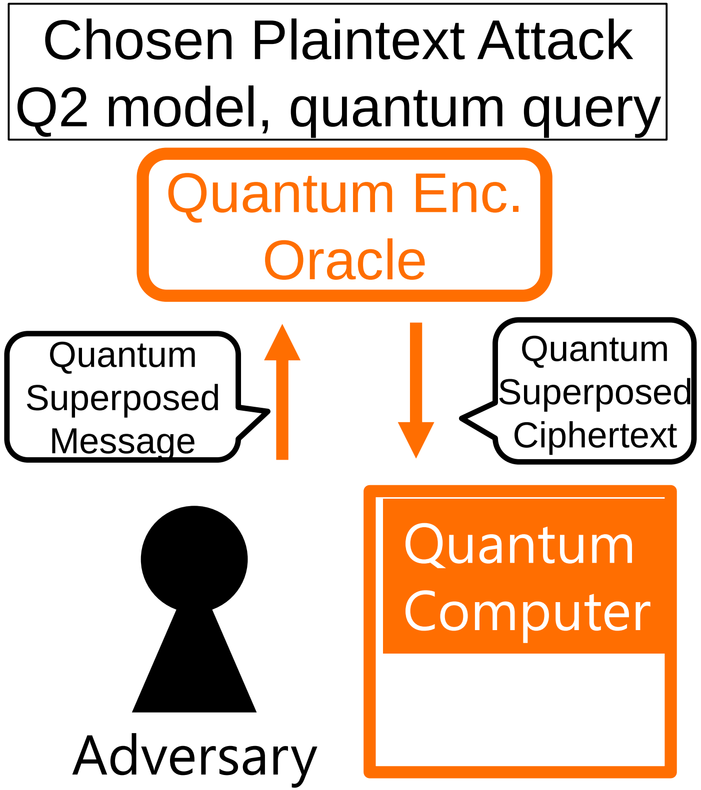
Attack Models





Attack Models





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Why should we consider quantum query attacks?

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Ex.) Offline Simon's algorithm at Asiacrypt 2019.

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Ex.) Offline Simon's algorithm at Asiacrypt 2019.

A3. For hash functions, quantum query attacks are natural

A4. If a scheme is secure against quantum query attacks, it can be used in cryptographic applications that run on quantum computers.

Quantum Query Attacks

Known Quantum Attacks : Today

NTT

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Simon's algorithm

Remark : The last two attacks assumes that quantum keyed oracles are available

Simon's period finding algorithm

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Problem

Suppose $f: \{0,1\}^n \to S$ and $s \in \{0,1\}^n$ satisfy

$$\forall x \in \{0,1\}^n \;\; f(x \oplus s) = f(x)$$

Given an oracle access to f, find s.

Classical algorithms: Exponential time

Simon's quantum algorithm: Polynomial time [Sim97]

[Sim97] Daniel R Simon. On the power of quantum computation. *SIAM journal on computing*, 26(5):1474–1483, 1997.

Simon's period finding algorithm

Problem

To mount poly-time attacks,
it is important to reduce
the target problem to Simon's problem

[Sim97] Daniel R Simon. On the power of quantum computation. *SIAM journal on computing*, 26(5):1474–1483, 1997.

Key-Recovery Attack on Even-Mansour

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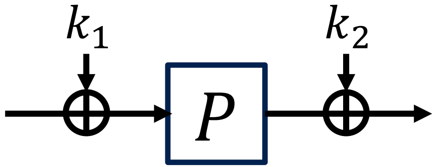
Even-Mansour cipher E_{k_1,k_2}

(P:public permutation)



Quantum CPA against Even-Mansour ciphers

$f(x) = E_{k_1,k_2}(x) \oplus P(x)$ **satisfies** $f(x \oplus k_1) = f(x)$

- **We <u>can recover k_1 in polynomial time</u> with Simon's algorithm**
- k_2 **can easily be recovered since we have**

$$E_{k_1,k_2}(x) \oplus P(x \oplus k_1) = k_2$$

[KM12] H. Kukakado and M. Morii: Security on the quantum-type Even-Mansour cipher. ISITA 2010.

Various MACs/AEs are broken in poly-time…

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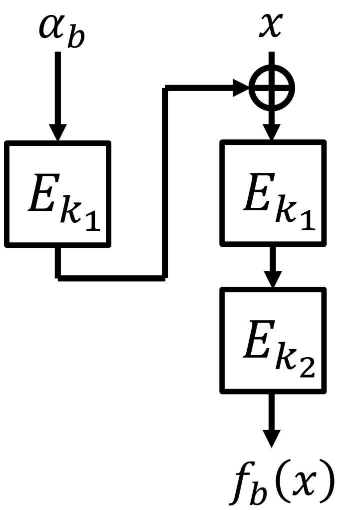
If quantum queries are allowed, Simon's algorithm breaks

- CBC-MAC
- PMAC
- GMAC
- GCM
- OCB

…



In polynomial time !

M. Kaplan, G. Leurent, A. Leverrier, and M. Naya-Plasencia: Breaking symmetric cryptosystems using quantum period finding (CRYPTO 2016)

Luby-Rackoff (Feistel) Construction

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Security in the classical setting

	PRP? (secure against CPA?)	SPRP? (secure against CCA?)
2-round	×	×
3-round	○	×
4-round	○	○
5-round	○	○

M. Luby, C. Rackoff: How to construct pseudo-random permutations from pseudorandom functions (CRYPTO '85)

Luby-Rackoff (Feistel) Construction

NTT

Security in the quantum setting

	PRP? (secure against CPA?)	SPRP? (secure against CCA?)
2-round	×	×
3-round	× [KM10]	×
4-round	O [HI19]	× [IHMSI19]
5-round	O [HI19]	?

[KM10] M. Luby, C. Rackoff: Quantum distinguisher between the 3-round Feistel cipher and the random permutation (ISIT 2010)
[IHMSI19] G. Ito, A. Hosoyamada, R. Matsumoto, Y. Sasaki, T. Iwata: quantum chosen-ciphertext attacks against Feistel ciphers? (CT-RSA 2019)
[HI19] A. Hosoyamada, T. Iwata: 4-Round Luby-Rackoff construction is a qPRP. (Asiacrypt 2019)

Other Quantum Query Attacks

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- Speed-up for differential/linear cryptanalysis [KLLN16b]
- Key recovery attacks on Feistel by using the quantum distinguishers [HS18b,IHMSI19]
- The attack with Kuperberg's algorithm [BN18]
- The attack on the FX construction by Leander and May [LM17]
- Speed-up for Demiric-Secluk meet-in-the-middle attack [HS18b, BNS19]

[BN18] X. Bonnetain, M. Naya-Plasencia: Hidden Shift Quantum Cryptanalysis and Implications, Asiacrypt 2018.

[HS18b] A. Hosoyamada, Y. Sasaki: Quantum Demiric-Selçuk Meet-in-the-Middle Attacks: Applications to 6-Round Generic Feistel Constructions, SCN 2018.

[IHMSI19] G. Ito, A. Hosoyamada, R. Matsumoto, Y. Sasaki, T. Iwata: Quantum Chosen-Ciphertext Attacks Against Feistel Ciphers. CT-RSA 2019.

[KLLN16b] M. Kaplan, G. Leurent, A. Leverrier, M. Naya-Plasencia: Quantum Differential and Linear Cryptanalysis. IACR Trans. Symmetric Cryptol. 2016(1), pp. 71-94.

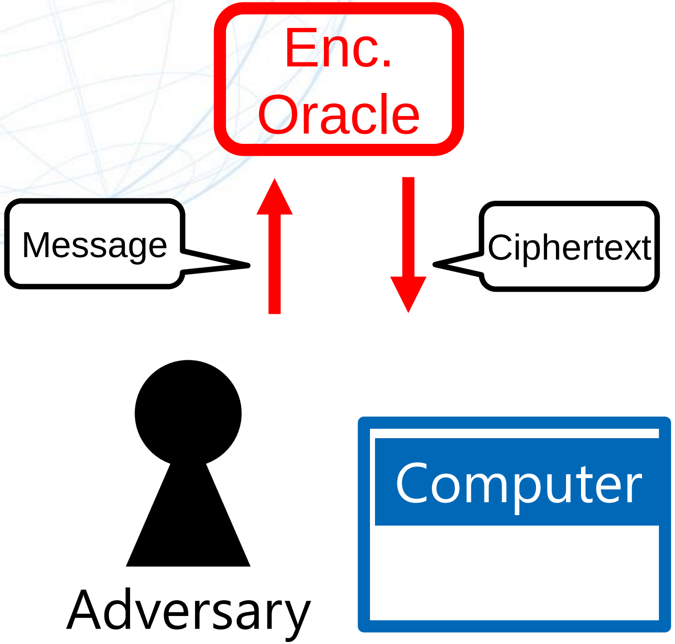
[LM17] G. Leander, A. May: Grover Meets Simon - Quantumly Attacking the FX-construction. Asiacrypt 2017.

[BNS19] X. Bonnetain, M. Naya-Plasencia, A. Schrottenloher: Quantum Security Analysis of AES. IACR Trans. Symmetric Cryptol. 2019(2), pp. 55-93.

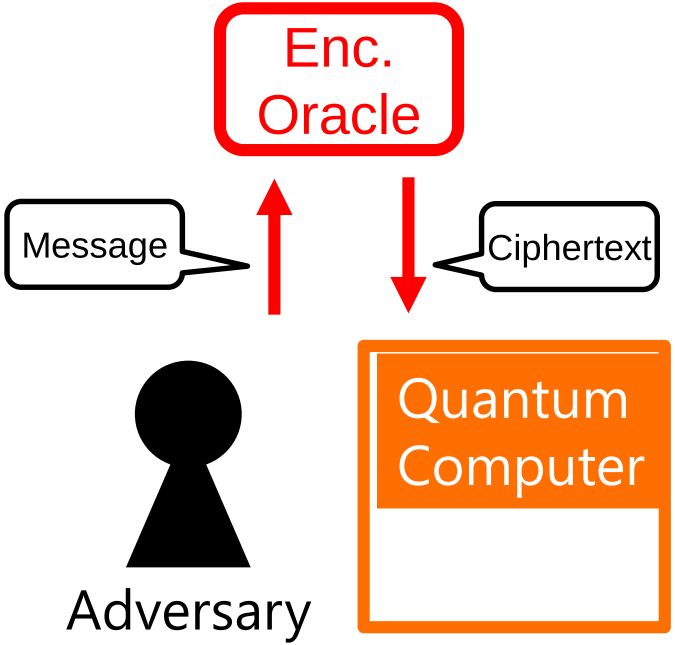
Attacks with Classical Query + Quantum Computation

Attack Models

Chosen Plaintext Attack



Chosen Plaintext Attack
Q1 model, classical query



Offline Simon’s algorithm (AC 2019)

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Quantum query attack with Simon’s algorithm is applicable

+

Simple On-Off MITM attack is applicable in the classical setting

⇓

Even if quantum queries are not allowed and just a small quantum computer is available, by using Simon’s algorithm we can mount a memory-efficient attack

X. Bonnetain, A. Hosoyamada, M. Naya-Plasencia, Y. Sasaki, A. Schrottenloher: Quantum Attacks without Superposition Queries: the Offline Simon’s Algorithm (Asiacrypt 2019)

Offline Simon's algorithm (AC 2019)

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(Q1 / Classical query) attacks on Even-Mansour

	Time	Query	Q. Mem	C. Mem
Kuwakado & Morii [KM12]	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$
Hosoyamada & Sasaki [HS18a]	$2^{3n/7}$	$2^{3n/7}$	Poly(n)	$2^{n/7}$
Offline Simon	$2^{n/3}$ $(< 2^{3n/7})$	$2^{n/3}$	**poly(n)**	**poly(n)**

Note: Polynomial factors are ignored. Only classical queries are allowed to keyed oracles.
No parallelized computations.

[KM12] H. Kukakado and M. Morii: Security on the quantum-type Even-Mansour cipher. ISITA 2010.
[HS18a] A. Hosoyamada, Y. Sasaki: Cryptanalysis Against Symmetric-Key Schemes with Online Classical Queries and Offline Quantum Computations, CT-RSA 2018.

Other classical query attacks

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- Differential / Linear Cryptanalysis [KLLN16b]
- Online-Offline meet-in-the-middle attacks [HS18a]
- Demiric-Selçuk meet-in-the-middle attacks [BNH19,HS18b]

and more…

[KLLN16b] M. Kaplan, G. Leurent, A. Leverrier, M. Naya-Plasencia: Quantum Differential and Linear Cryptanalysis. IACR Trans. Symmetric Cryptol. 2016(1), pp. 71-94.

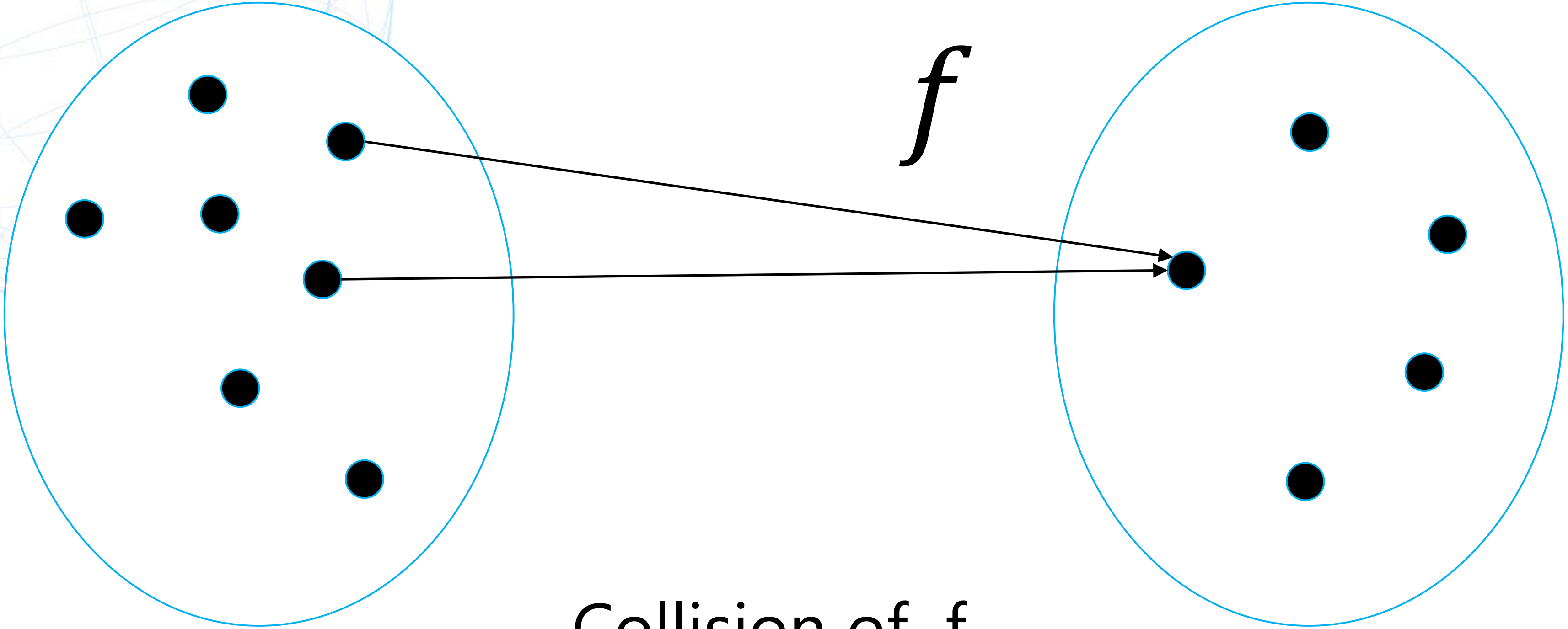
[BNS19] X. Bonnetain, M. Maya-Plasencia, A. Schrottenloher: Quantum Security Analyais of AES. IACR Toransactoins on Symmetric Cryptology, 2019(2).

[HS18a] A. Hosoyamada, Y. Sasaki: Cryptanalysis Against Symmetric-Key Schemes with Online Classical Queries and Offline Quantum Computations, CT-RSA 2018.

[HS18b] A. Hosoyamada, Y. Sasaki: Quantum Demiric-Selçuk Meet-in-the-Middle Attacks: Applications to 6-Round Generic Feistel Constructions, SCN 2018.

Generic Attacks on Hash

Collision Attack on Hash



Collision of f

Collision Attack on Hash

The number of queries required to find a collison

Classical : $\Theta(N^{1/2})$

Quantum : $\Theta(N^{1/3})$

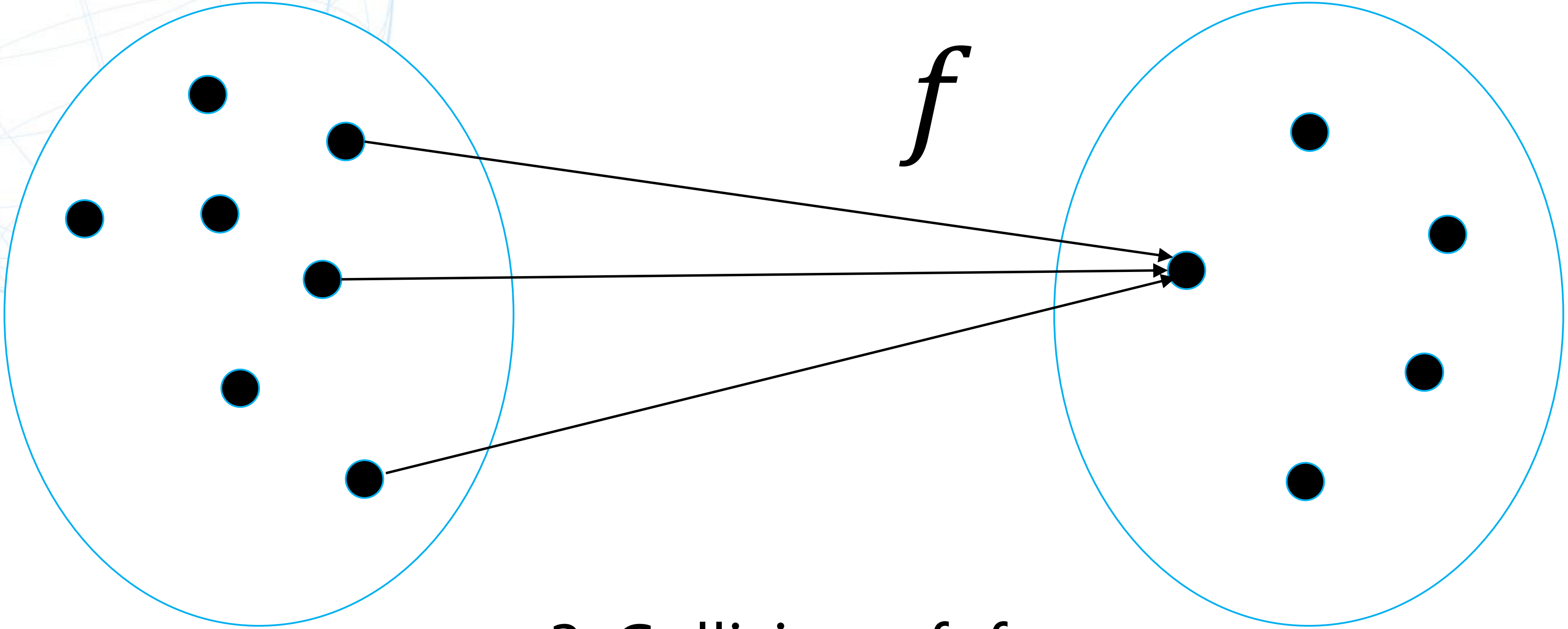
[BHT97,Zha15]

The BHT Algorithm

[BHT97] G. Brassard, P. Hoyer, A. Tapp: Quantum cryptanalysis of hash and claw-free functions. ACM Sigact News, 28(2), pp. 14-17 (1997).

[Zha15] M. Zhandry: A note on the quantum collision and set equality problems. Quantum Information & Computation 15(7&8): pp. 557-567 (2015)

MultiCollision Attack on Hash



3-Collision of f

MultiCollision Attack on Hash

NTT

The number of queries required to find an ℓ-collison

Classical : $\Theta(N^{(\ell-1)/\ell})$ [STKT08]

Quantum : $\Theta\left(N^{\frac{2^{\ell-1}-1}{2^{\ell}-1}}\right)$

[HSX17, HSTX19,LZ19]

[STKT08] K. Suzuki, D. Tonien, K. Kurosawa, K. Toyota : Birthday paradox for multi-collisions. IEICE Transactions, 91-A(1):39–45, 2008

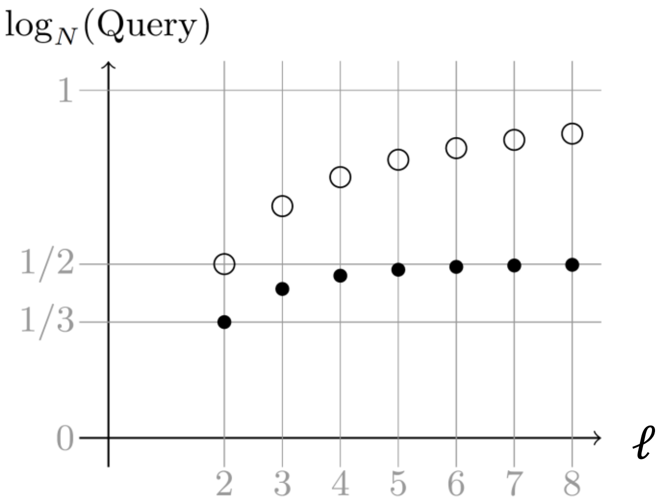
[HSX17] A. Hosoyamada, Yu Sasaki, K. Xagawa: Quantum Multicollision-Finding Algorithm. Asiacrypt 2017.

[HSTX19] A. Hosoyamada, Yu Sasaki, S. Tani, K. Xagawa: Improved Quantum Multicollision-Finding Algorithm. PQCrypto 2019.

[LZ19] Q. Liu, M. Zhandry: On Finding Quantum Multi-collisions, Eurocrypt 2019.

MultiCollision Attack on Hash

ℓ (multiplicity)	2	3	4	5
Classical (○)	$N^{\frac{1}{2}}$	$N^{\frac{2}{3}}$	$(N^{\frac{3}{4}})$	$(N^{\frac{4}{5}})$
Quantum (●)	$N^{\frac{1}{3}}$	$N^{\frac{3}{7}}$	$N^{\frac{15}{31}}$	$N^{\frac{31}{63}}$



Other generic attacks on hash

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- Collision finding with polynomial number of qubits[CNS17]
  - The BHT algorithm finds a collision in time $\boldsymbol{N^{1/3}}$ but requires $\boldsymbol{N^{1/3}}$ qubits…
  - Even if only poly-qubits are available, collision can be found in time $\boldsymbol{N^{2/5}}$ $(< \boldsymbol{N^{1/2}})$
- Acceleration for the k-xor problem[Amb07, GNS18]
- Multi-target preimage search [BB17, CNS17]
  - Applicable to key recovery in multi-key/user setting

[Amb07] Quantum walk algorithm for element distinctness. SIAM J. Comput. 37(1), 210-239 (2007).
[BB17] G. Banegas, D. Bernstein: Low-Communication Parallel Quantum Multi-Target Preimage Search. SAC 2017.
[CNS17] A. Chailloux, M. Naya-Plasencia, A. Schrottenloher: An Efficient Quantum Collision Search Algorithm and Implications on Symmetric Cryptography. Asiacrypt 2017.
[GNS18] L. Grassi, M. Naya-Plasencia, A. Schrottenloher: Quantum Algorithms for the k-xor Problem. Asiacrypt 2018.

Challenges for the future in cryptanalysis

Attacks on keyed primitives

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- More attacks on concrete primitives
- Applications of quantum algorithms other than Simon (period finding), Grover, Quantum-walk-search
- New quantum algorithms (attacks) that are specific to concrete symmetric key schemes
- Other applications of quantum algorithms in the classical query model

and more…

Generic attacks on hash

- New Time-Memory tradeoff for inverting functions that is better than the classical tradeoff?

Time-Memory tradeoff for inverting function

- f: random function/permutation (n-bit to n-bit) / $\mathcal{A}$: adversary

1. $\mathcal{A}$ runs precomputation with h and store (classical/quantum) data of size S
2. $\mathcal{A}$ receives a randomly chosen y
3. $\mathcal{A}$ tries to find x s.t. f(x) = y in time T by using the stored data

Time-Memory tradeoff for inverting function

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Classical tradeoff between T and S:

$T = 2^n/S$ (if f is a random permutation)

Time-Memory tradeoff for inverting function

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<u>Classical tradeoff between T and S</u>:

$T = 2^n/S$ (if f is a random permutation)

<u>Quantum tradeoff between T and S</u>:

So far, there does not exist any tradeoff that is better than $T = 2^n/S$

Grover search achieves $T = 2^{n/2}$ when S=1 but it is not clear what kind of trade-off is possible when S > 1…

Security Proofs / Lower bounds

What has already been done?

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Generic bounds on random functions (query complexity)

- Preimages of random functions: $\Theta(N) \rightarrow \Theta\left(N^{1/2}\right)$
- RP-RF switch: $\Theta\left(N^{1/2}\right) \rightarrow \Theta\left(N^{1/3}\right)$
- Multicollision-Finding problem: $\Theta\left(N^{\frac{(\ell-1)}{\ell}}\right) \rightarrow \Theta\left(N^{\frac{2^{\ell-1}-1}{2^{\ell}-1}}\right)$
- k-xor: $\Theta\left(N^{\frac{1}{k}}\right) \rightarrow \Theta\left(N^{\frac{1}{k+1}}\right)$

Red: Classical Bound

Blue: Quantum Bound

What has already been done?

Security proofs for specific schemes

(against quantum query attacks, w/o algebraic assumptions)

- CPA security of encryption modes (CTR, CBC, OFB,…) (@PQCrypto2016)
- Generic composition for AE (@PQCrypto2016)
- PRF security of NMAC/HMAC (@CRYPTO2017)
- Sponge-like construction
  - PRF security of sponge with keyed (secret) permutation (@CRYPTO2017)
  - Collision-resistance (collapsing) of sponge with public function (@PQCrypto2018)
- Indifferentiability of (fixed-length) Merkle-Damgaard (@CRYPTO2019)
- PRP security of 4-Round Luby-Rackoff (Feistel) (@Asiacrypt 2019)

What is difficult in the quantum setting?

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1. It is not trivial how to record queries
   - Copying the values of queries disturbs the adversary's quantum states, which leads to changing its behavior significantly

2. "Lazy Sampling" is not available
   - In classical proofs, the value F(x) of a random function F is randomly chosen on the fly when the adversary queries x to F
   - At most one value is fixed per each classical query
   - In the quantum setting, the adversary may query a superposition of all possible x at the same time…

The Compressed Oracle Technique

Compressed Oracle Technique [Zha19]

- It enables us to do "Lazy sampling" to some extent for random functions in the quantum setting
- The important observation: Sometimes recorded information should be "forgotten"
- Many applications:

  Quantum Indifferentiability of Merkle-Damgaard[Zha19]

  Lower bound for multicollision finding problem[LZ19]

  quantum PRP security of 4-round Luby-Rackoff[IH19]

  etc···

[Zha19] M. Zhandry: How to record quantum queries, and applications to quantum indifferentiability. Crypto 2019.

[LZ19] Q. Liu, M. Zhandry: On Finding Quantum Multi-collisions, Eurocrypt 2019.

[IH19] A. Hosoyamada, T. Iwata: 4-round Luby-Rackoff Construction is a qPRP. Asiacrypt 2019.

The Compressed Oracle Technique

One remark:

Zhandry's compressed oracle technique cannot be applied to permutations

Remarks on query lower bound

Research Area	Problems	Backward query?
Quantum computation	Worst case	×
Public key crypto	Average case (randomized)	×
Symmetric key crypto	Average case (randomized)	○

Remarks on query lower bound

Research Area	Problems	Backward query?
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Symmetric key crypto	Average case (randomized)	○

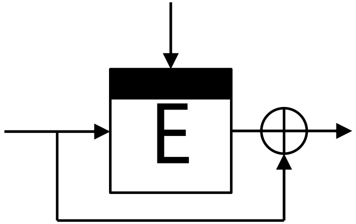
It is hard to treat permutations…

- So far there is no published results on quantum proof techniques for public random permutation or ideal cipher

- Exception: One-wayness of Davies-Meyer Compression function

  [HY18]

  - Giving security proofs by computing statistical distance
  - (so far & as far as I know) the only published results on quantum proofs for schemes in ideal permutation model / ideal cipher model w/o algebraic assumptions



[HY18] A. Hosoyamada, K. Yasuda: Building quantum one-way functions from block ciphers: Davies-Meyer and Merkle-Damgaard constructions. Asiacrypt 2018.

Challenges for the future

NTT

- Generic and strong proof technique to treat random permutations / ideal ciphers
  - The compressed oracle technique: Since F is a random function, F(x) and F(y) are independent, which means that the quantum registers for F(x) and F(y) are not entangled
  - If we try to apply the compressed oracle technique to a random permutation P, P(x) and P(y) are ***not*** independent, which means that the quantum registers for P(x) and P(y) will be ***entangled***

Quantum entanglement always make things extremely difficult…

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Quantum entanglement always make things extremely difficult…

Solved??

Czajkowski, Majenz, Schaffner, Zur: Quantum lazy sampling and game-playing proofs for quantum indifferentiability. (ePrint 2019/428)

Summary

Summary

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- Recent results show many unexpected attacks are possible in the quantum setting
  - Many schemes are broken in poly-time with quantum queries
  - Simon's algorithm is applicable even if only classical queries are allowed
  - Various new tradeoffs
- There are lots of challenging but interesting topics to study
  - Time-memory tradeoffs for inverting functions?
  - Proof techniques for permutations?
  - AES can be broken with quantum algorithms?

Thank you!