Beyond Birthday Bound Security of CLRW1^4

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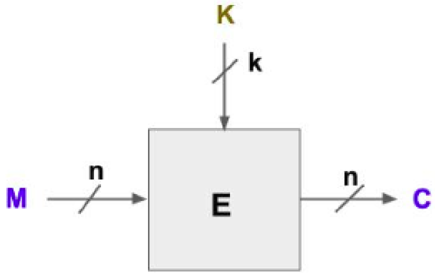
Inventing Harmonious Future

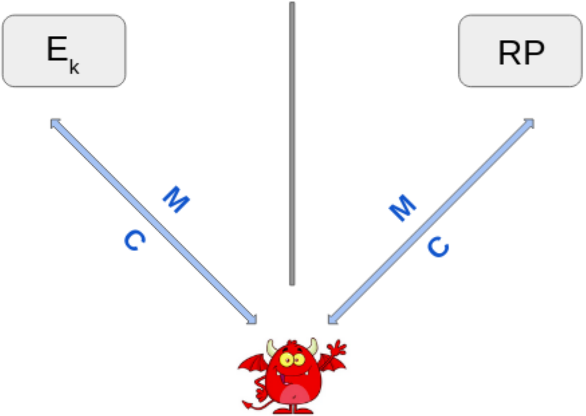
December 02, 2023

Block Cipher



- A family of permutations indexed by key
- Process fixed size data
- Key size and data size need not be equal
- For each key k, E_k is a permutation over $\{0,1\}^n$
- Popular examples of block ciphers are AES, PRESENT, GIFT etc.

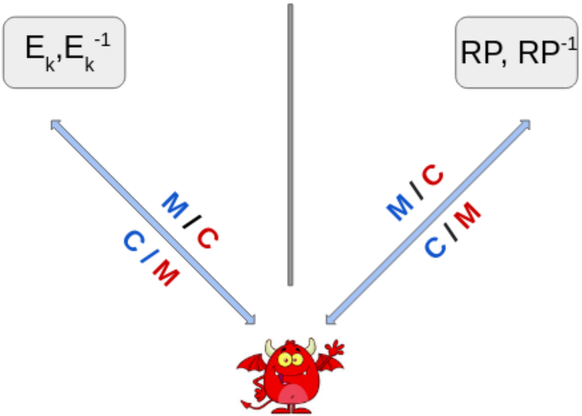
PRP Security :



$$\mathbf{Adv}_{\mathsf{E}}^{\mathrm{PRP}}(A) := |\Pr_{K \leftarrow \{0,1\}^k}[A^{\mathsf{E}_K} = 1] - \Pr_{\mathsf{RP} \leftarrow \mathsf{Perm}(n)}[A^{\mathsf{RP}} = 1]|$$

Formal Security Notion of BC

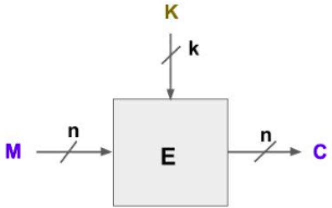
SPRP Security :



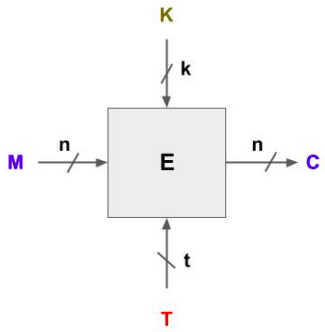
$$\mathbf{Adv}_{\mathsf{E}}^{\mathrm{SPRP}}(A) := \left| \Pr_{K \leftarrow \{0,1\}^k}[A^{\mathsf{E}_K, \mathsf{E}_K^{-1}} = 1] - \Pr_{\mathsf{RP} \leftarrow \mathsf{Perm}(n)}[A^{\mathsf{RP}, \mathsf{RP}^{-1}} = 1] \right|$$

Block Cipher in Modes of Operations

- Block Cipher processes fixed size data
- To process arbitrary size data, block ciphers are invoked as primitive in certain way that defines the modes of operations
- Although it provides security, but does not bring variability into the cipher
- To introduce variability, one needs to change the block cipher key
- Frequnt changes of key is a costly business



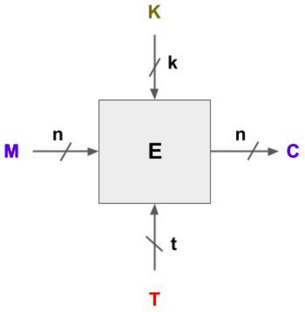
Tweakable Block Cipher



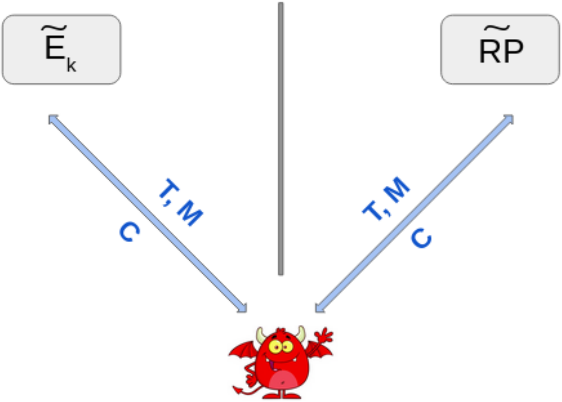
- An additional public value tweak T (controlled by adversary)
- Like Block Cipher, it process fixed size data
- For each (k, t), $M \mapsto \mathsf{E}_k^t(M)$ is a permutation over $\{0,1\}^n$

Tweakable Block Cipher

- Each fixed setting of the tweak gives rise to a independent family of BC
- Fixing the input and varying tweaks yields a random function.
- Fixing the tweak and varying input yields a random permutation
- Key provides uncertainity
- Tweak Provides variability

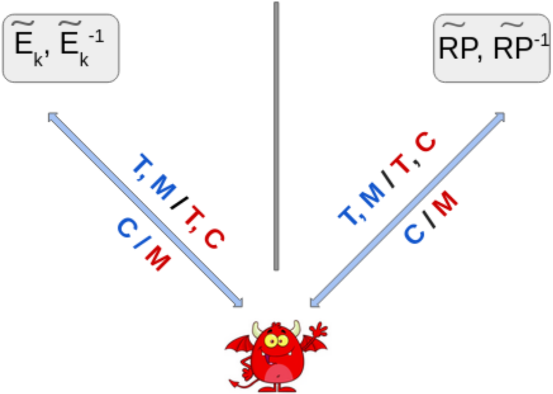


TPRP Security :



$$\mathbf{Adv}^{\mathrm{TPRP}}_{\widetilde{\mathsf{E}}}(A) := |\Pr_{K \leftarrow \{0,1\}^k}[A^{\widetilde{\mathsf{E}}_K} = 1] - \Pr_{\widetilde{\mathsf{RP}} \leftarrow \widetilde{\mathsf{Perm}}(n)}[A^{\widetilde{\mathsf{RP}}} = 1]|$$

STPRP Security :



$$\mathbf{Adv}_{\widetilde{\mathsf{E}}}^{\mathrm{STPRP}}(A) := \left| \Pr_{K \leftarrow \{0,1\}^k}\left[A^{\widetilde{\mathsf{E}}_K, \widetilde{\mathsf{E}}_K^{-1}} = 1\right] - \Pr_{\widetilde{\mathsf{RP}} \leftarrow \widetilde{\mathsf{Perm}}(n)}\left[A^{\widetilde{\mathsf{RP}}, \widetilde{\mathsf{RP}}^{-1}} = 1\right] \right|$$

Designing Tweakable Block Ciphers

- There are two ways to design a tweakable block cipher:
  1. Design from classical block ciphers in a black box fashion
  2. Design tweakable block ciphers from scratch

- In the black box setting, there are two design approaches:
  1. Tweakable block ciphers are designed from classical block ciphers by assuming that the underlying block ciphers are pseudorandom permutations.
  2. This design strategy was introduced by Liskov et al.

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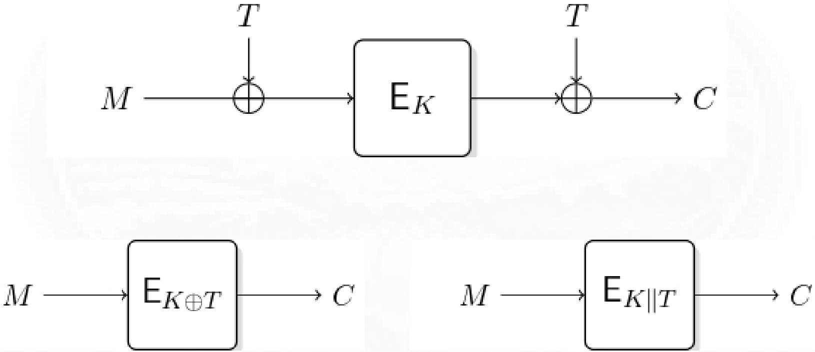
  2. This design strategy was introduced by Mennink.

- These two design approaches not only differ in their security assumptions but also in their design principles.

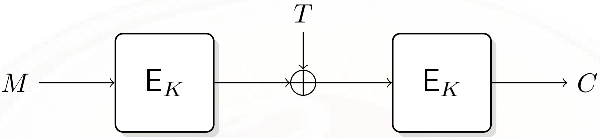
Our study is on designing TBC from BC where BC is a PRP

Designing Tweakable Block Ciphers from Block Ciphers

Some Insecure Constructions:



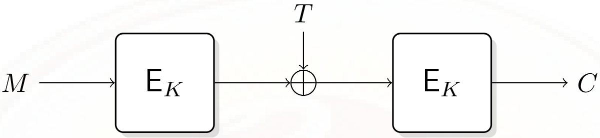
Designing Tweakable Block Ciphers from Block Ciphers



LRW1 Construction, [Liskov et al., CRYPTO'02]

Tight CPA security upto $2^{n/2}$ queries, assuming E_k is secure PRP

Designing Tweakable Block Ciphers from Block Ciphers



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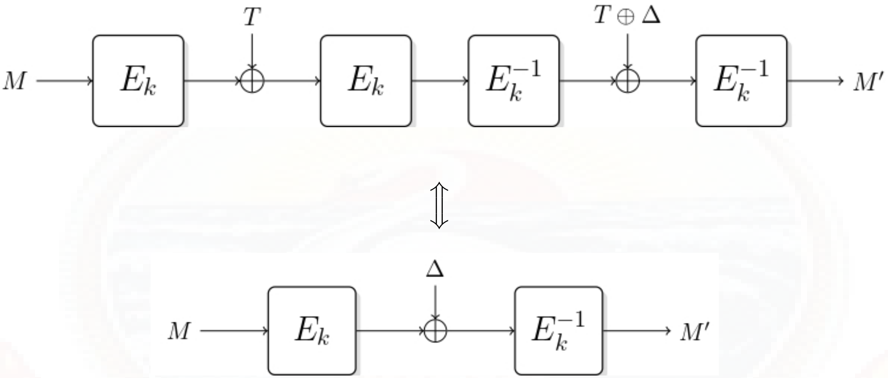
LRW1 is not CCA secure

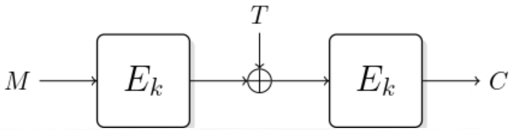
CCA Insecurity of LRW1 Construction



Characteristic Equation: $E_K(M) \oplus E_K(M') = \Delta$

CCA Insecurity of LRW1 Construction

Attack Algorithm



Adversary $\mathcal{A}$ makes an encryption query (M, T) and obtains the ciphertext C

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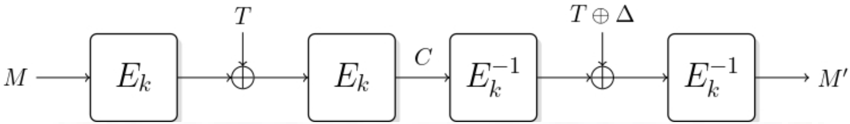

Adversary $\mathcal{A}$ makes a decryption query $(C, T \oplus \Delta)$ and obtains the plaintext M'

(I) It yields the characteristic equation: $E_K(M) \oplus E_K(M') = \Delta$

CCA Insecurity of LRW1 Construction

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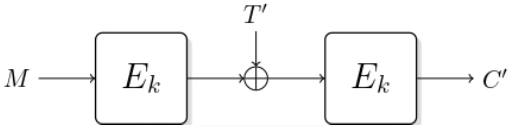
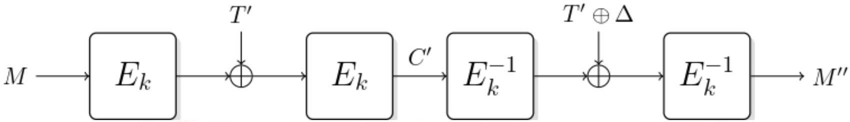


Adversary $\mathcal{A}$ makes another encryption query (M, T') and obtains the ciphertext C'

Attack Algorithm



Adversary $\mathcal{A}$ makes a decryption query $(C', T' \oplus \Delta)$ and obtains the plaintext M''

(II) It yields the characteristic equation: $E_K(M) \oplus E_K(M'') = \Delta$

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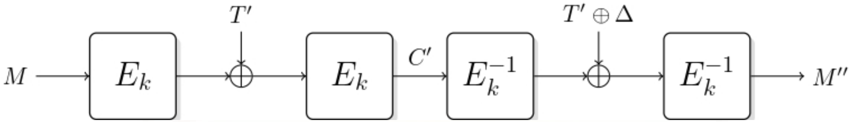

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CCA Insecurity of LRW1 Construction

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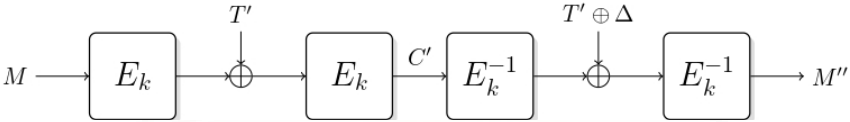

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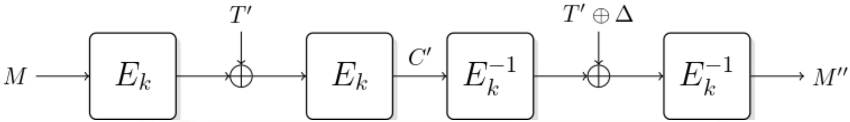

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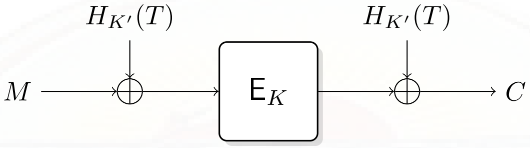
- Can we have a STPRP secure TBC with a single BC call ?
- How many BC calls are required to yield STPRP security with linear tweak mixing ?

Designing Tweakable Block Ciphers from Block Ciphers



LRW2 Construction, [Liskov et al., CRYPTO'02]

Tight CCA security upto $2^{n/2}$ queries, assuming E_k is SPRP and H is AXU

Follow-ups on LRW2 Construction

- Extended LRW2 to achieve BBB security [**Landecker et al., CRYPTO'12**]
  - Two-round cascading of LRW2 (CLRW2) is secure upto $2^{2n/3}$ queries
- Extended cascading LRW2 to r round [**Lampe and Seurin, FSE'13**]
  - CLRW2^r achieves CCA security upto $2^{(r-1)n/(r+1)}$ queries, when r is odd
  - CLRW2^r achieves CCA security upto $2^{rn/(r+2)}$ queries, when r is even
- Improved CLRW2 to a tight $3n/4$-bit CCA security bound [**Mennink, TCC'18 and Jha, Nandi, J. Cryptol.'20**]

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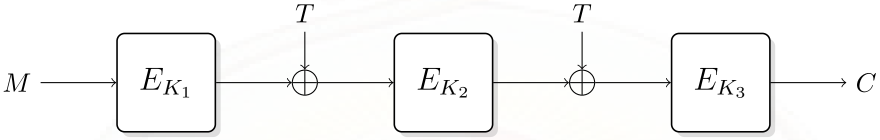
Such kind of follow up works is absent for LRW1 construction until the work of [Bao et al., EC'20]

Recent Developments on LRW1



CLRW1^3 Construction, [Bao et al., EC'20]

- CLRW1^3 achieves CCA security upto $2^{2n/3}$ many queries **[Bao et al., EC'20]**

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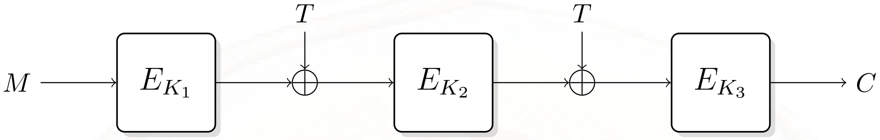
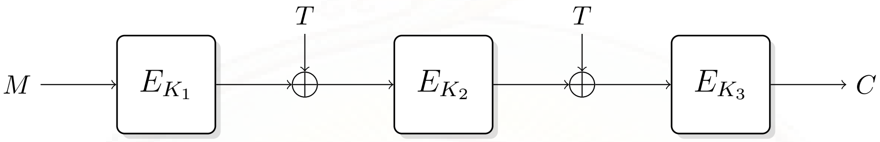


CLRW1^3 Construction, [Bao et al., EC'20]

- CLRW1^3 achieves CCA security upto $2^{2n/3}$ many queries **[Bao et al., EC'20]**
- CLRW1^3 achieves tight CPA security upto $2^{3n/4}$ queries **[Guo et al., AC'20]**



CLRW1^3 Construction, [Bao et al., EC'20]

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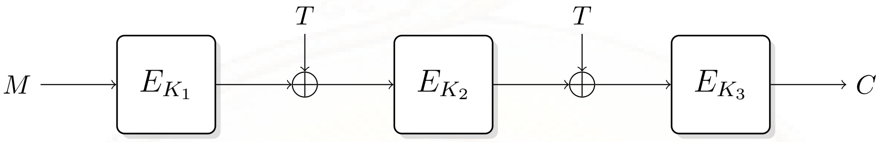

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- CLRW1^r achieves CCA security upto $2^{(r-2)n/r}$ many queries, when r is even **[Zhang et al. DCC'22]**

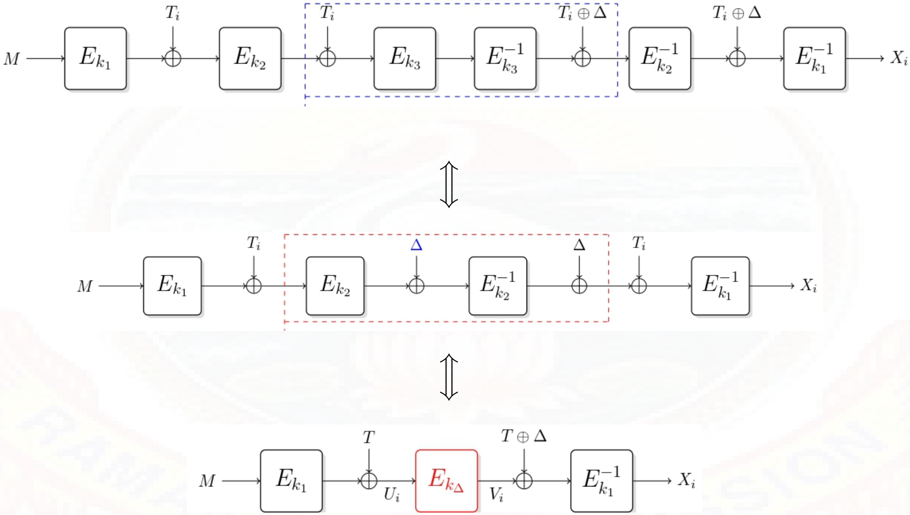
Invalid Security Bound of Bao et al. EUROCRYPT'20

- Extended cascading LRW1 to r round [**Zhang et al., DCC'22**]
  - $\mathsf{CLRW1}^r$ achieves CCA security upto $2^{\frac{(r-1)n}{r+1}}$ many queries, when r is odd
  - $\mathsf{CLRW1}^r$ achieves CCA security upto $2^{\frac{(r-2)n}{r}}$ many queries, when r is even

- Presented a birthday bound CCA distinguishing attack on $\mathsf{CLRW1}^3$ [**Khairallah, ePrint Arch., 2023/1233**]
  - Security claim of **Bao et al.** stands invalid

Birthday Bound Attack on CLRW1[3]

Extension of the CCA Attack on LRW1



Birthday Bound Attack on CLRW1^3

- The attack was first demonstrated by **[Khairallah et al., eprint 2023/1233]**.
- The attack was based on the statistics of random permutation
- Later, **[Jha et al., eprint 2023/1272]** presented a distinguishing attack and analyzed the its success probability in a more formal way

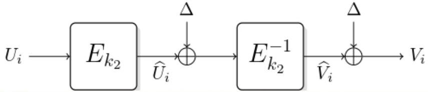
We describe the attack and analysis by [Jha et al., eprint 2023/1272]

Birthday Bound Attack on CLRW1[3]

- Fix a message $m \in \{0,1\}^n$
- Fix a subspace $\mathcal{T} = \{t_1, t_2, \ldots, t_q\} \subseteq \{0,1\}^n$.
- Fix a $\Delta \notin \mathcal{T}$.
- For all $t_i \in \mathcal{T}$, do the following:

- Make encryption query (m, t_i) and the response is C_i
- Make the decryption query $(C_i, t_i \oplus \Delta)$ and the response is X_i
- $\mathcal{A}$ outputs 1 if $\exists j < i$ such that $X_i = X_j$

Ideal World Collision Probability: $\Pr[\exists i \neq j : X_i = X_j] = 1 - \frac{(2^n)_q}{2^{nq}}$



Real World Collision Probability:

- $X_i = X_j \Leftrightarrow V_i \oplus V_j = t_i \oplus t_j$
- Since the same message m is used in the encryption, it implies $U_i \oplus U_j = t_i \oplus t_j$
- Therefore, $X_i = X_j \Leftrightarrow U_i \oplus V_i = U_j \oplus V_j$
- $X_i = X_j \Leftrightarrow U_i \oplus \widehat{U}_i = V_j \oplus \widehat{V}_j$

Birthday Bound Attack on CLRW1[3]

- $E_0 := \exists i \neq j : U_i \oplus V_i = U_j \oplus V_j$ holds if and only if
  1. $E_1 := \exists i \neq j : \widehat{U}_i \oplus \widehat{U}_j = \Delta$ or
  2. $E_2 := \exists i \neq j : (E_{k_2}^{-1}(\widehat{U}_i \oplus \Delta) \oplus U_i = E_{k_2}^{-1}(\widehat{U}_j \oplus \Delta) \oplus U_j)$
- Therefore, $E_0 \Leftrightarrow E_1 \vee E_2$

$$\Pr[E_0] = \Pr[E_1] + \Pr[E_1^c \wedge E_2]$$

- It is easy to calculate that $\Pr[E_1] \geq (2^n)_q / 2^{nq}$

$$\Pr_R[\exists i \neq j : X_i = X_j] = \Pr_I[\exists i \neq j : X_i = X_j] + \Pr[E_1^c \wedge E_2]$$

Birthday Bound Attack on CLRW1^3

$$\mathbf{Adv}^{tsprp}_{\mathsf{CLRW1}^3} \geq \Pr[E_1^c \wedge E_2] = \Pr[E_1^c]\Pr[E_2|E_1^c]$$

- It can be shown that

$$\Pr[E_1^c] \geq \left(1 - \frac{(2^n)_q}{2^{nq}}\right) \cdot \left(1 - \frac{2q^3}{2^{2n}}\right).$$

- Using simple algebra, one can show that

$$\Pr[E_2|E_1^c] \geq \alpha(q)\left(1 - \frac{\alpha(q)}{2}\left(1 + \frac{2}{2^n - q - 3}\right)\right).$$

- where $\alpha(q) = \binom{q}{2}/(2^n - q - 1)$

$$\mathbf{Adv}^{tsprp}_{\mathsf{CLRW1}^3} \geq \alpha(q)(1 - \alpha(q))(1 - \tfrac{\alpha(q)}{2} - \tfrac{\alpha(q)}{2^n - q - 3})\left(1 - \tfrac{2q^3}{2^{2n}}\right)$$

Current Status

- CLRW1^3 achieves tight BB CCA security
  - Birthday bound CCA attack on CLRW1^3 due to **[Khairallah, eprint 2023/1233]** and **[Jha et al., eprint 2023/1272]**
  - Tightness of the bound is due to **[Zhang et al., DCC'22]**, and **[Jha et al., eprint 2023/1272]**
- CLRW1^4 achieves BB CCA security due to **[Zhang et al., DCC'22]**
- CLRW1^5 achieves BBB CCA security due to **[Zhang et al., DCC'22]**
- However, the bound of **[Zhang et al., DCC'22]** is loose due to the application of coupling lemma.

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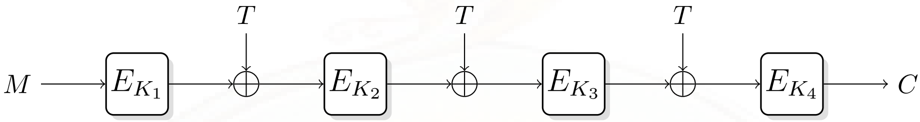
> How many rounds are required for CLRW1 to achieve BBB security against all adaptive CCA adversaries?

4 Rounds Cascading of LRW1



CLRW1^4 Construction

- **We have shown CLRW1^4 is secure upto $2^{\frac{3n}{4}}$ CCA queries**
- **Hence, four rounds are sufficient for CLRW1 to achieve BBB security**

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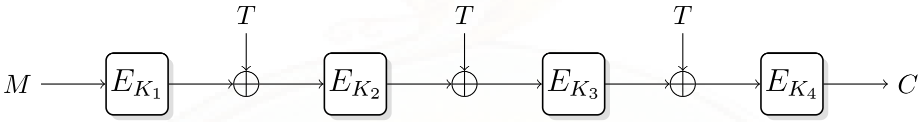

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- **Hence, four rounds are sufficient for CLRW1 to achieve BBB security**

† Concurrent to this work, **[Jha et al., eprint 2023/1272]** have also shown $3n/4$ bit security of CLRW1^4

Security Result

Suppose,

- Block cipher $E : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$
- $\mathcal{A}$: An (q,t) adversary against the strong tweakable pseudo random permutation security of CLRW1^4 ($q \leq 2^{\frac{3n}{4}}$)

Then,

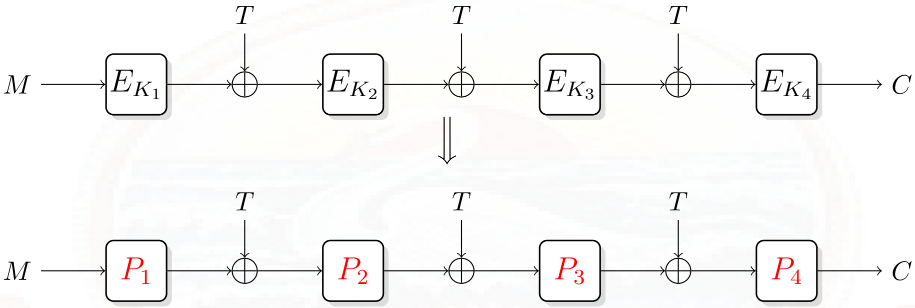
- $\exists \mathcal{A}'$: An (q,t') adversary against the strong pseudo random permutation security of E ($t = t'$)

such that

$$\textbf{Adv}^{\text{tsprp}}_{\text{CLRW1}^4[E]}(\mathcal{A}) \leq 4\textbf{Adv}^{\text{sprp}}_{E}(\mathcal{A}') + \frac{6q^2}{2^{2n}} + \frac{4q^{\frac{4}{3}}}{2^n} + \frac{38q^4}{2^{3n}}$$

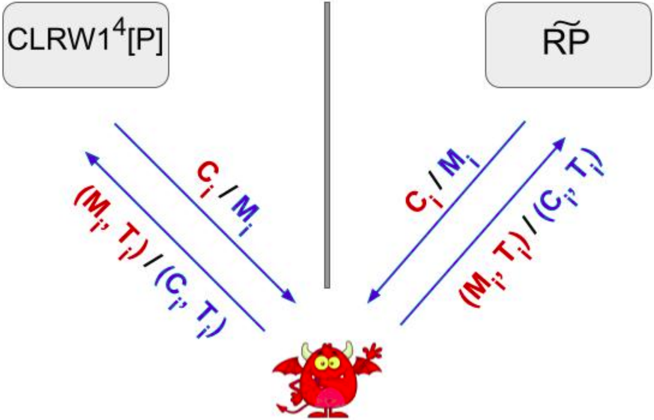
Sketch of the proof

Step 1: Replace Block Cipher with Random Permutation



$$\mathbf{Adv}^{\text{tsprp}}_{\mathsf{CLRW1}^4[E]}(\mathcal{A}) \leq 4\mathbf{Adv}^{\text{sprp}}_{E}(\mathcal{A}') + \mathbf{Adv}^{\text{tsprp}}_{\mathsf{CLRW1}^4[P]}(\mathcal{A})$$

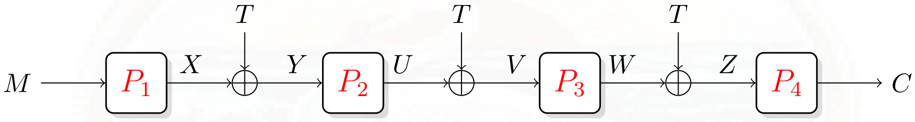
Online Phase of the Interaction:



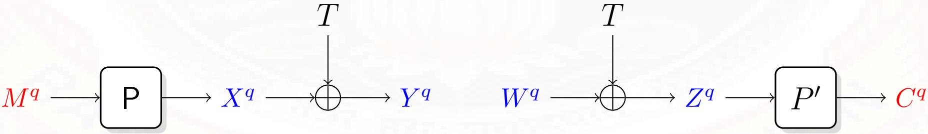
Sketch of the proof: Releasing Intermediate Variables

It reveals the intermediate variables (X, Y, U, V, W, Z)

Releasing in Real World



Releasing in the Ideal World

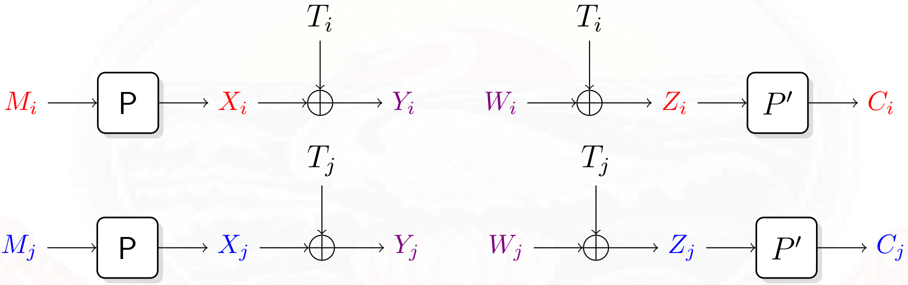


$(\mathbf{U^q}, \mathbf{V^q})$ is yet to be released in the ideal world.

Sketch of the proof: Identifying Bad Events

Partial Trascript: $(M^q, X^q, Y^q, W^q, Z^q, C^q)$

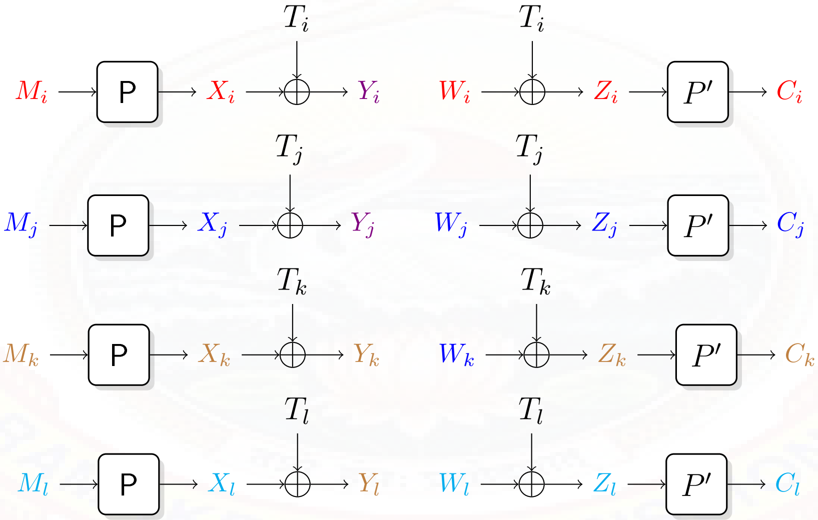
Bad 1: $\exists i, j \in [q]$ such that $Y_i = Y_j, W_i = W_j$



- **Bad 2:** $\left|\{(i,j) \in [q]^2 : Y_i = Y_j\}\right| \geq q^{\frac{2}{3}}$
- **Bad 3:** $\left|\{(i,j) \in [q]^2 : W_i = W_j\}\right| \geq q^{\frac{2}{3}}$

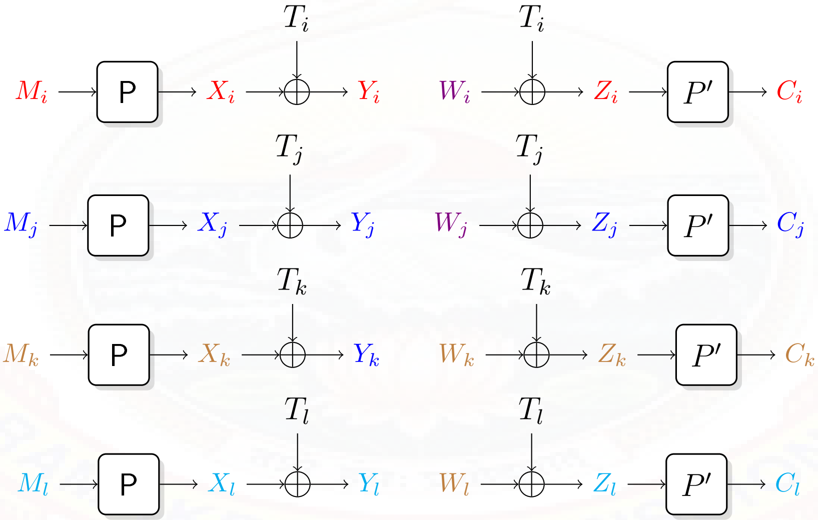
Sketch of the proof: Identifying Bad Events

Bad 4: $\exists i, j, k, l \in [q]$ such that $Y_i = Y_j, W_j = W_k, Y_k = Y_l$



Sketch of the proof: Identifying Bad Events

Bad 5: $\exists i, j, k, l \in [q]$ such that $W_i = W_j, Y_j = Y_k, W_k = W_l$



Sketch of the proof: Sampling (U^q, V^q) in the Ideal World

We construct an edge labeled bipartite graph given the partial transcript is **NOT BAD**

- **Vertices:** $\mathcal{V}_1 = \{Y_1, Y_2, \cdots, Y_q\} \bigcup \mathcal{V}_2 = \{W_1, W_2, \cdots W_q\}$
- **Labeled Edges:** $\{Y_i, W_i\} \in E$ with label T_i

Merge Y_i and Y_j if $Y_i = Y_j$ and W_i and W_j if $W_i = W_j$

Sketch of the proof

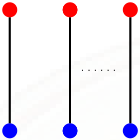


Figure: Type-I

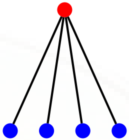


Figure: Type-II

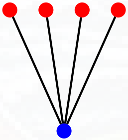


Figure: Type-III

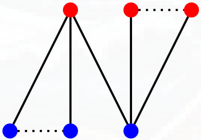


Figure: Type-IV

Sketch of the proof: Sampling (U^q, V^q)

- Consider $\mathcal{I} = \mathcal{I}_1 \sqcup \mathcal{I}_2 \sqcup \mathcal{I}_3$, where $\mathcal{I}_b = \{i \in [q] : (Y_i, W_i) \in \text{Type}_b\}$
- Consider $\mathcal{E} := \left\{U_i \oplus V_i = T_i : i \in \mathcal{I}\right\}$
- Solution set, $\mathcal{S} = \left\{(U_i, V_i) : U^{\mathcal{I}} \leftrightsquigarrow Y^{\mathcal{I}}, V^{\mathcal{I}} \leftrightsquigarrow W^{\mathcal{I}}, U_{\mathcal{I}} \oplus V_{\mathcal{I}} = T_{\mathcal{I}}\right\}$
- Sample $(U^{\mathcal{I}}, V^{\mathcal{I}}) \xleftarrow{\$} \mathcal{S}$

However, it remains to sample (U, V) for Type-IV component

- Select (Y_i, W_i) such that $\text{deg}(Y_i) = \text{deg}(W_i) \geq 2$
- Sample $U_i \xleftarrow{\$} \{0, 1\}^n$
- Set $V_i = U_i \oplus T_i$

Sketch of the proof: Sampling (U^q, V^q)

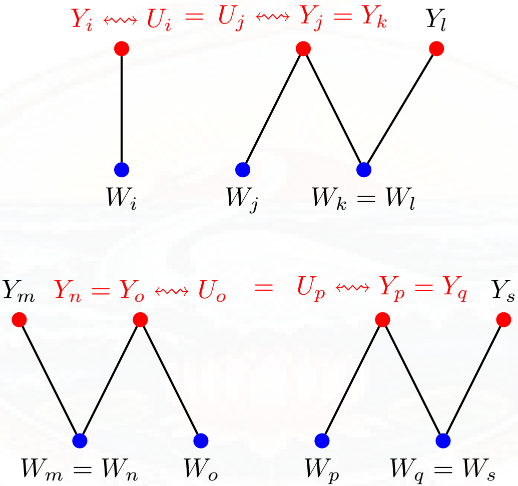
The sampling may lead to permutation incompatible transcript

Sampling Induced Bad Events

- $\mathsf{Ucoll}_{\alpha\beta}$: $\exists i \in \mathcal{I}_\alpha,\ j \in \mathcal{I}_\beta$ such that $Y_i \neq Y_j$ and $U_i = U_j$
- $\mathsf{VColl}_{\alpha\beta}$: $\exists i \in \mathcal{I}_\alpha,\ j \in \mathcal{I}_\beta$ such that $W_i \neq W_j$ and $V_i = V_j$

Bad-samp $:= \bigcup_{\substack{\alpha \in [4] \\ \beta \in [\alpha,4]}} (\mathsf{UColl}_{\alpha,\beta} \cup \mathsf{VColl}_{\alpha,\beta})$

Sketch of the proof: Bad Sampling



Sketch of the proof: Analysis of Good Transcripts

Real World: Count the number of times each permutation is invoked

Ideal World:

- **Type-1, type-2, type-3 Sampling:**
  - Non-degenerate
  - Does not contain any cycle
  - Maximum component size $\leq q^{\frac{2}{3}}$
- Used Mirror Theory results for the tweakable permutation for Type-I, Type-II, and Type-III
- We sample (U, V) in consistent way for Type-IV component and bound the ideal interpolation probability.

Open Problems

1. Is the proven security bound for CLRW1^4 tight or not?
2. Whether the bounds of CLRW1^r for general $r \geq 5$ can be improved.
3. CPA bound of three-round CLRW1 is $3n/4$-bit secure. It is interesting to see whether we can prove the CPA bound of four round CLRW1 upto $4n/5$ bits.

Joint Work with Nilanjan Datta, Shreya Dey and Sougata Mandal.
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Thank You!